Superposition

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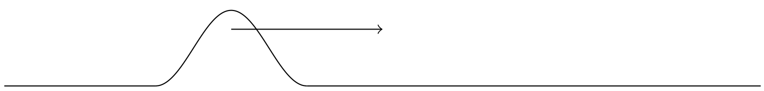
Superposition

We have already seen that a progressive wave is a pattern of displacement which moves along with time. What happens, then, when more than one wave is present (this can happen for any medium, e.g. on a string or on water)?

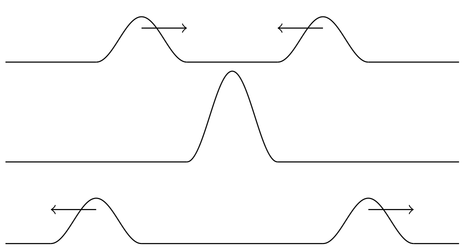
The displacement pattern with more than one wave can be worked out using the *principle of superposition*. The principle of superposition states that whenever more than one pattern of displacement (wave) is present, the total displacement is the sum of the individual displacements.

To see how this works in more detail, we are first going to consider some wave *pulses* (a pulse is a single disturbance in a medium, e.g. by flicking a rope at one end up and back again only once).

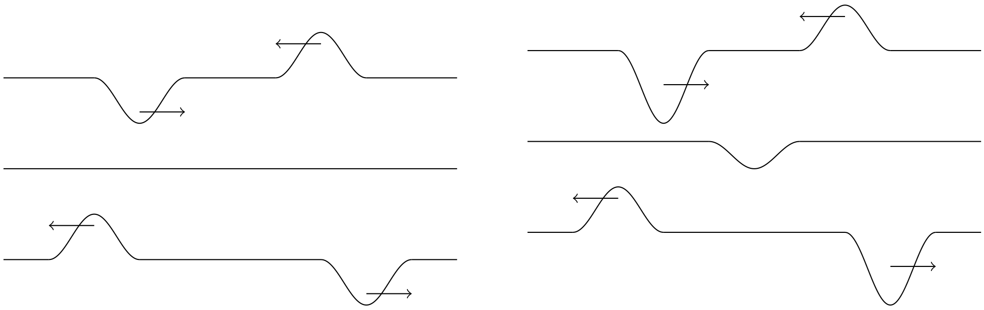


Two or more pulses can pass through a medium at one time, and the principle of superposition enables us to find out what will happen by adding their individual effects. After they pass through each other, they continue in the same direction, and their original amplitudes remain unchanged.

Constructive superposition is when the total effect of two pulses is greater when they meet each other. They superimpose to create a larger pulse when they are in the same place at the point of meeting, and the amplitude of the resulting pulse is equal to the sum of the amplitudes of the initial pulses.



Destructive superposition is when the total effect is smaller when pulses meet each other. They superimpose to partly or totally cancel out. The amplitude of the resulting pattern is still the sum of the combining amplitudes, but one of those amplitudes will be negative.



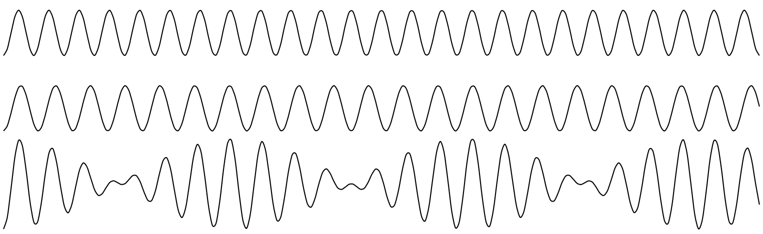
Superposition of continuous waves

Superposition can occur with continuous waves as well as simple pulses. The principle of superposition is applied in exactly the same way to these situations (i.e. we add up the displacements from the contributing waves at each point to find out the resulting wave pattern).

The waves don't have to be travelling in opposite directions towards each other like the pulses above; they may be travelling in the same direction (or any direction like ripples on water), or there may be many different waves all with different frequencies, amplitudes and directions, all of which can be superimposed using the same rule.

Beats

Below is the result of combining the waves from two sources of slightly different frequencies:



The resulting wave has a higher frequency than either of the original waves[1], and keeps getting louder and quieter. The loudness changes because as the waves superimpose constructively and destructively in turn: since they have almost the same frequency, the waves will be 'in sync' for a time (their

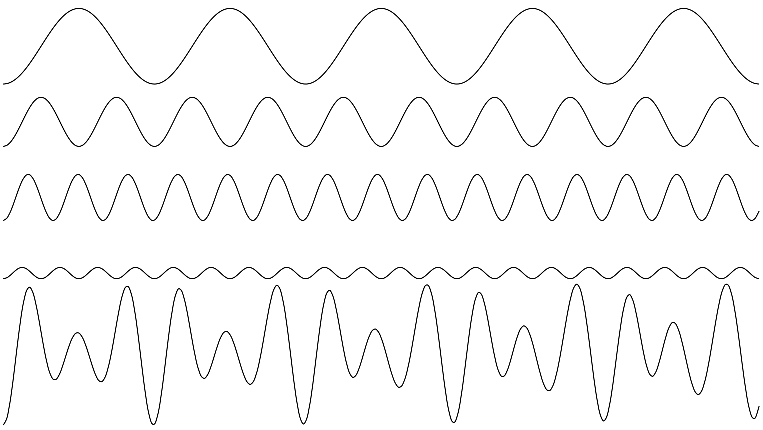
[1] In fact, the sum of the two frequencies.

crests and troughs will coincide so they will make a bigger wave), before they go 'out of sync' and will cancel each other out, and this happens repeatedly due to their frequency difference[2].

These repeated variations in the loudness of the resulting wave can easily be heard in sound waves where two sounds of nearly the same pitch are heard at the same time and are known as *beats*. The beats get slower as the frequencies get closer, and disappear altogether when the frequencies are identical, so this is an important method of tuning musical instruments (a piano tuner has actually trained to listen for these beats in order to temper a piano's tuning).

Harmonics

As we shall see in the following section on standing waves, musical instruments often produce *harmonics*, which are sound waves of certain higher frequencies than the note they are playing, in addition to that note. These harmonics supeimpose onto each other, and give the waveform of the musical instrument its shape (which gives each musical instrument its distinctive sound or tone quality). The example below shows the four main waves for a flute playing a B at 247 Hz, and its final waveform (the amplitudes are accurate.[3])



[2]The frequency of these loudness variations (or amplitude modulations if you prefer) is in fact the difference in frequencies of the contributing waves.

[3]Source: `http://hyperphysics.phy-astr.gsu.edu/hbase/music/flutew.html`