## The diffraction grating

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A diffraction grating is a sheet of glass, usually about $4\,\mathrm{cm}$ square, with many thousands of very narrow parallel lines etched onto one side. These lines make slits with a width of typically have two or three times the wavelength of visible light (meaning that light passing through each individual slit is akin to sound passing through a doorway).

When a parallel beam of monochromatic light encounters a diffraction grating at normal incidence, the light beam is split up beyond the grating into a number of transmitted beams, which can go on to form images called *principal maxima*. This is due to the superposition of the diffracted light wave patterns from each of the slits in the grating, which leads to their interfering destructvely everywhere beyond the grating except in certain specific directions; in only these directions do they reinforce each other to give a light beam. One of the directions in which light is transmitted is the normal direction (i.e. the same direction as the incident beam), because here the path differences for pairs of slits at equal distance from the centre of the beam are zero, and so these will constructively interfere.
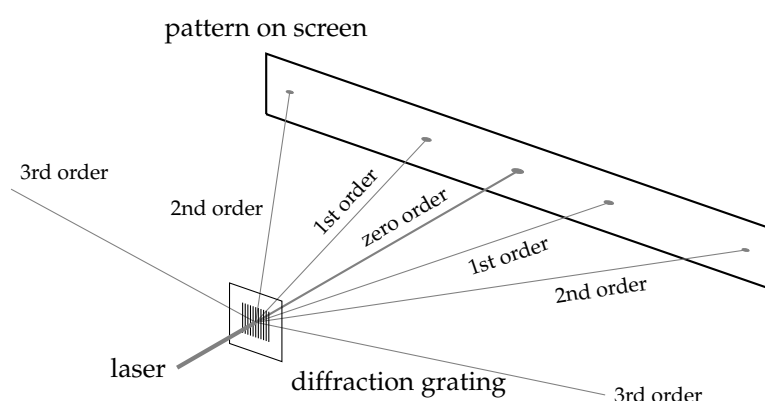
The transmitted beams are conventionally given an order number n outwards from the central 'zero order' beam, and the angle of diffraction θ_n is also measured round from the zero order beam to each transmitted beam. This angle increases (i.e. the beam spots are more spaced out on the screen) as the line spacing of the diffraction grating is decreased, or equivalently as the wavelength of the light is increased.

In general, if λ is the wavelength of the light transmitted, and d is the distance between centres of adjacent slits (as in the experiment shown above), the angle θ_n between the nth order beam and the zeroth order beam is given[1] by

$$d \sin \theta_n = n\lambda.$$

We can derive the diffraction grating equation by considering two light rays which are diffracted from adjacent slits. In order to interfere constructively, these diffracted rays must be in phase with

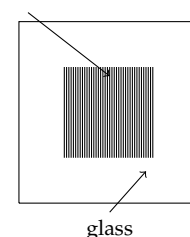about 500 lines per mm, all parallel and equidistant

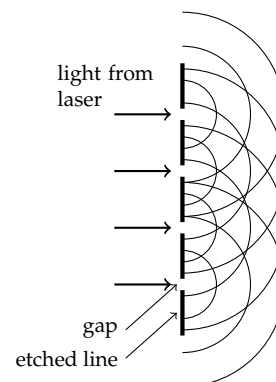Figure 1: Note that lines on a diffraction grating cannot usually be seen.

Figure 2: The diffraction from the grating's slits is extreme—each one is effectively a source of circular waves—and the principle of superposition is needed to decide which directions the light will take).

[1] Very often the spacing of gratings is given as the number of lines per metre (lines/mm and even l.p.i.=lines per inch are also common in laboratories), and since $d = \frac{1}{N}$, where N is the number of lines per metre,

$$\sin \theta_n = Nn\lambda.$$
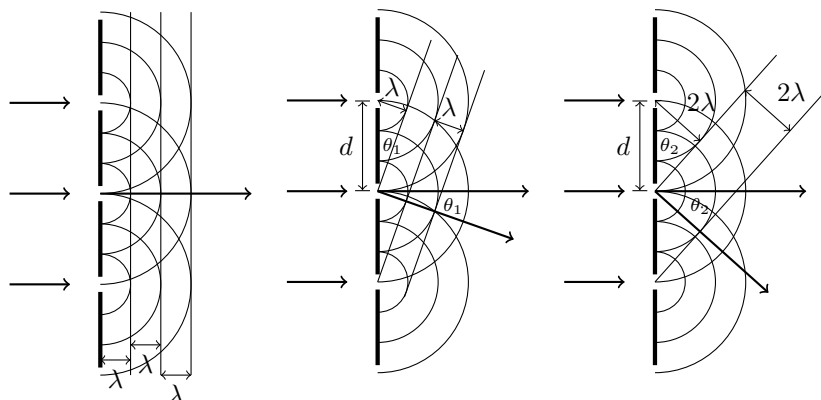
each other, meaning that the path difference between their path lengths must be a whole number of wavelengths of the light $n\lambda$:

$$d \sin \theta_n = n\lambda$$
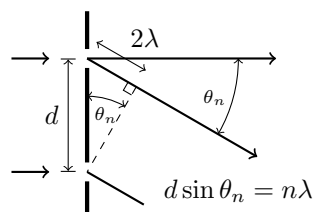
Since the slits have equal spacing d, if the light from adjacent slits is in phase, the light from each slit is in phase with that from any other. The effect of this is to produce bright images—the principal maxima—and it can also be shown that provided the number of slits is large, completely destructive interference is produced at other angles, so diffraction gratings give rise to very narrow diffracted beams, and sharp principal maxima.

Applications of the diffraction grating

Diffraction gratings can be used in a spectrometer, a device which can measure wavelengths very accurately and can be used to study the spectrum of light from any light source. A common use is to determine the wavelength of lines in a line spectrum, arising from energy level transitions of electrons inside the atom.

The angle at which any of the principal maxima is formed (except the $n = 0$ maximum) depends on the wavelength of the light, and so a grating illuminated by white light produces a series of spectra, with violet being deviated less than red light (note that this is opposite to the behaviour of light dispersed by a prism, which also only produces one spectrum and has a different angular spacing of colours). Since each wavelength produces a maximum at $\theta = 0$, the central maximum is white.
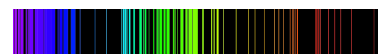

Figure 3: Iron emission spectrum, showing the many spectral lines. Since each element has a characteristic 'fingerprint' of spectral lines, this information can be used to determine the elements present and their proportions (for example in light from a flame test or light from a star).