RADLEY

The Stefan–Boltzmann Law

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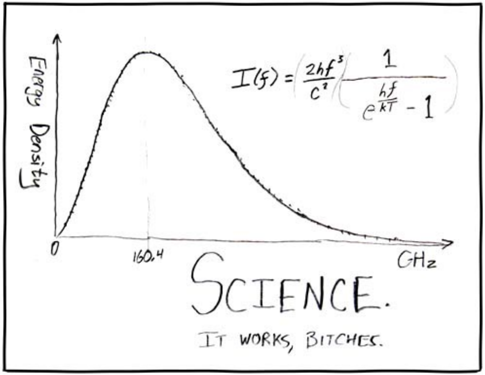
Quantum mechanics ($\hbar$) explains surface temperatures of planets & stars



https://xkcd.com/54/

Today we shall

1. see how to derive the Stefan–Boltzmann law.
2. know some applications of the law.
3. be able to use the law in calculations and to answer questions.

Textbook p. 119 [APFY]

Specification Requirement

7 Using radiation to investigate stars

(b) the idea that bodies which absorb all incident radiation are known as black bodies and that stars are very good approximations to black bodies

(d) . . . Stefan's law and the inverse square law to investigate the properties of stars – luminosity, size, temperature and distance [NB stellar brightness in magnitudes will not be required]

[Eduqas A Level Physics Specification, 2009/10 onwards]

Start with relevant quantities

F	MT^{-3}	energy flux
$k_B T$	$\mathrm{ML}^2\mathrm{T}^{-2}$	thermal energy
$\hbar$	$\mathrm{ML}^2\mathrm{T}^{-1}$	quantum
c	LT^{-1}	speed of light

Now we could use algebra to make $F \times (k_B T)^x \hbar^y c^z$ dimensionless (this would be messy).

More physically meaningful to make $c \equiv 1$ and $\hbar \equiv 1$

- Choosing $c \equiv 1$ expresses unity of space and time (Einstein: SR), and makes length and time have the same units.
- Choosing $c \equiv 1$ also expresses equivalence of mass and energy (Einstein: SR), and makes mass and energy have the same units.
- Choosing $\hbar \equiv 1$ expresses the fundamental insight of quantum physics: that energy is (angular) frequency:

$$E = \hbar\omega.$$

When $\hbar \equiv 1$, then E is ω.

These choices of $c \equiv 1$ and $\hbar \equiv 1$ also affect dimensions

- Choosing $c \equiv 1$ makes dimensions of length and time equivalent:

$$c \equiv 1 \text{ implies } \mathrm{L} \equiv \mathrm{T}.$$

  Thus, in our table, we can replace T with L.

- Choosing $\hbar \equiv 1$ contributes the dimensional equation

$$\underbrace{\mathrm{ML}^2\mathrm{T}^{-2}}_{[E]} \equiv \underbrace{\mathrm{T}^{-1}}_{[\omega]}.$$

  This looks like a mess, but replacing T with L (from $c \equiv 1$) simplifies it:

These choices of $c \equiv 1$ and $\hbar \equiv 1$ also affect dimensions

Length and time are equivalent dimensions.

Mass and inverse length are equivalent dimensions.

Now we'll rewrite our table of dimensions

F	energy flux
$k_B T$	thermal energy
$\hbar$	quantum
c	speed of light

Now we'll rewrite our table of dimensions

Energy flux, F

F	M^4	energy flux
$k_B T$		thermal energy
$\hbar$		quantum
c		speed of light

- In the usual system, dimensions were MT^{-3}.
- Now, T^{-3} is equivalent to M^3.
- The dimensions of F become M^4.

Now we'll rewrite our table of dimensions

Thermal energy, $k_B T$

F	M^4	energy flux
$k_B T$	M	thermal energy
$\hbar$		quantum
c		speed of light

- In the usual system, dimensions were $\mathrm{ML}^2\mathrm{T}^{-2}$.
- Now, T^{-2} is equivalent to M^2.
- Dimensions of $k_B T$ become M (they should!)

Now we'll rewrite our table of dimensions

F	M^4	energy flux
$k_B T$	M	thermal energy
c		speed of light

Quantum constant, $\hbar$

- By fiat (when we choose $\hbar \equiv 1$), is now dimensionless.
- We do not list it.

Now we'll rewrite our table of dimensions

F	M^4	energy flux
$k_B T$	M	thermal energy

Speed of light, c

- Also by fiat, $c \equiv 1$.
- We do not list it.

Now we'll rewrite our table of dimensions

F	M^4	energy flux
$k_B T$	M	thermal energy

Because F contains M^4 and $k_B T$ contains M, the form of the relationship between them must be

$$F \sim (k_B T)^4 .$$

No lunch is free, and no good deed goes unpunished!

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$$F = \underbrace{\frac{\pi^2}{60}\frac{{k_B}^4}{\hbar^3 c^2}}_{\sigma} T^4$$

$$\sigma \equiv \frac{\pi^2}{60}\frac{{k_B}^4}{\hbar^3 c^2}$$

Calculate the Sun's surface temperature, T_{Sun}

Hints: use the Stefan–Boltzmann law, find the energy flux at the Sun's surface, F_{Sun}, using proportional reasoning ($F_{\text{Earth}} = 1300\,\text{W/m}^2$).

Prep

Use the Stefan–Boltzmann law to predict the surface temperature of the Earth. Your prediction ought to be somewhat colder than reality. How do you explain the (life-saving) difference between prediction and reality?

Lightbulb filament