RADLEY

# Flux

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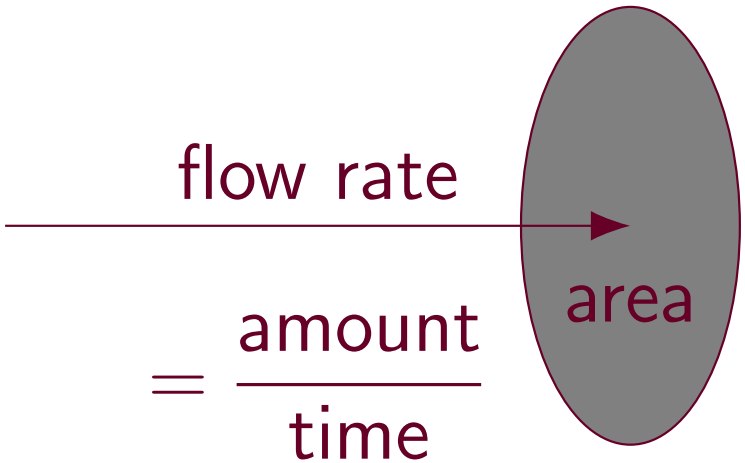
# Physical flows live in a geometry

Physical flows are an amount per time (rate):




$$= \frac{\text{amount}}{\text{time}}$$

The geometry allows us to define a related quantity:

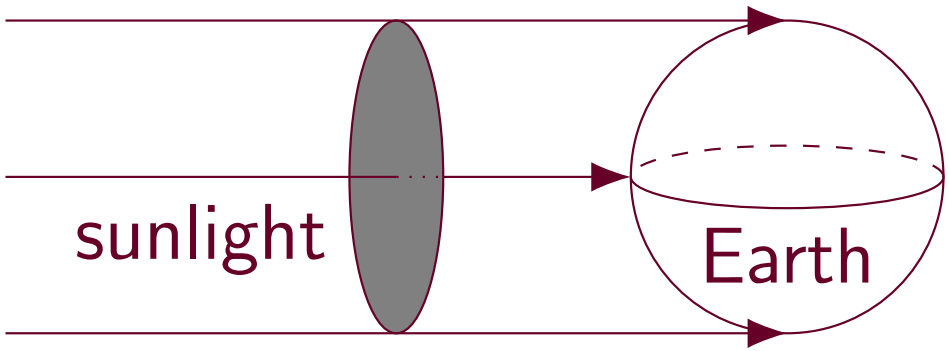
$$\text{flux of stuff} \equiv \frac{\text{rate}}{\text{area}} = \frac{\text{amount of stuff}}{\text{area} \times \text{time}}$$

# Flux thoughts

1. Explain why rate (amount per time) is more useful than amount.
2. What kind of flux (flux of what?) is current density?

# What is the average solar flux on Earth's surface?

At top of atmosphere, looking directly towards the Sun, the flux is $1300\,\mathrm{W/m^2}$.



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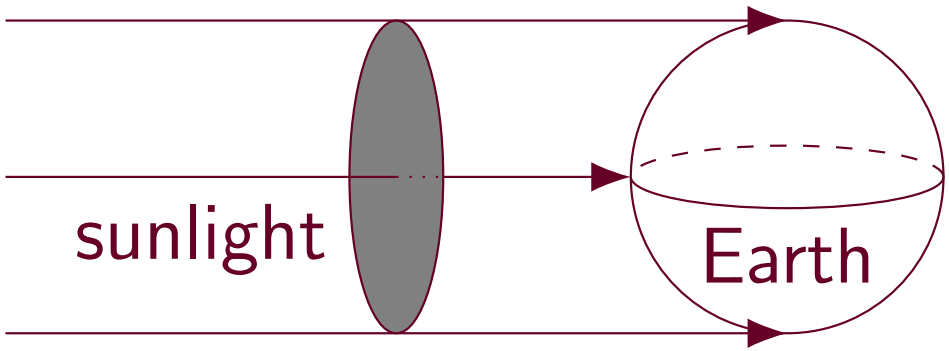


$$\text{power} = \text{energy flux} \times \text{area} = F\pi {R_{\text{Earth}}}^2.$$

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Remember that flux is connected to area, because flux is a rate per area.
Use *proportional reasoning*!

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Use *proportional reasoning*!

The solar flux $F$ at a distance $r$ from the Sun is the solar luminosity $L_{\text{Sun}}$—the radiant power output of the Sun—spread over a sphere with radius $r$:

$$F = \frac{L_{\text{Sun}}}{4\pi r^2}.$$

$r_{\text{Pluto}} = 40 \times r_{\text{Earth}}$

# What is the solar flux at Pluto's orbit?

What not to do:

$$r_{\text{Earth}} = \sqrt{\frac{L_{\text{Sun}}}{4\pi F_{\text{Earth}}}}\,\text{m} \quad \dashrightarrow \quad r_{\text{Pluto}} = 40 \times \sqrt{\frac{L_{\text{Sun}}}{4\pi F_{\text{Earth}}}}\,\text{m}$$

*(extra baggage)*

$$r = \sqrt{\frac{L_{\text{Sun}}}{4\pi r}} \qquad F = \frac{L_{\text{Sun}}}{4\pi r^2}, \quad F_{\text{Pluto}} = \frac{L_{\text{Sun}}}{4\pi {r_{\text{Pluto}}}^2}$$

$$F_{\text{Earth}} = 1300\,\text{W/m}^2 \xrightarrow[\text{prop. reasoning}]{F \propto r^{-2}} F_{\text{Pluto}} = \frac{1300}{40^2}\,\text{W/m}^2$$

# What is the solar flux at Pluto's orbit?

Instead, using *proportional reasoning*:

- As $r$ changes, $L_{\text{Sun}}$ and factor of $4\pi$ stay the same.
- Simplify $F = L_{\text{Sun}}/4\pi r^2$ to $F \propto r^{-2}$.
- This *scaling relation* is shorthand for

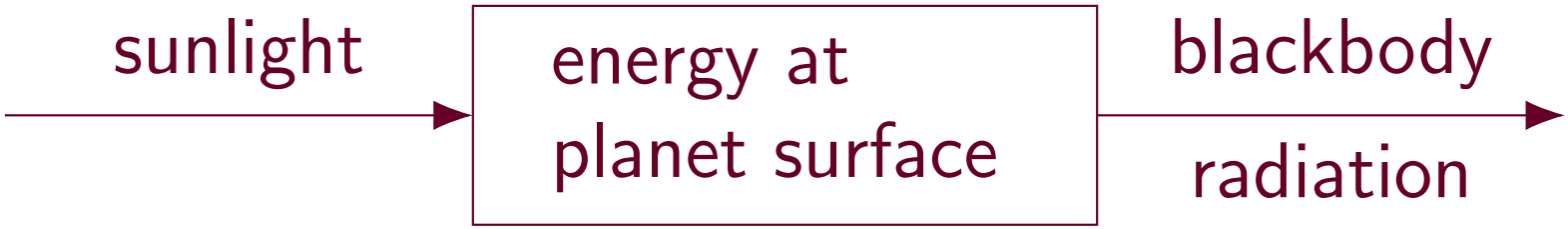
$$\frac{F_{\text{Pluto}}}{F_{\text{Earth}}} = \left(\frac{r_{\text{Pluto}}}{r_{\text{Earth}}}\right)^{-2}, \text{ or } F_{\text{Pluto}} = F_{\text{Earth}} \left(\frac{r_{\text{Pluto}}}{r_{\text{Earth}}}\right)^{-2}.$$

- Since $r_{\text{Pluto}} = 40 \times r_{\text{Earth}}$,

$$F_{\text{Pluto}} = \frac{1300\,\text{W}}{\text{m}^2} \times \frac{1}{1600} \approx \frac{0.8\,\text{W}}{\text{m}^2}.$$

# What is Pluto's surface temperature?

Surface temp. depends mostly on so-called blackbody radiation




Radiated flux $F$ is given by:

$$F = \sigma T^4,$$

where $\sigma \approx 5.7 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}^4}$.

# What is Pluto's surface temperature?

$$F = \frac{L_{\text{Sun}}}{4\pi r^2} \longrightarrow \boxed{\text{Pluto surface energy}} \longrightarrow F = \sigma T^4$$

Simplify to scaling relations:

$$F \propto r^{-2} \qquad\qquad F \propto T^4$$

Combine to a single scaling relation:

$r^{-2} \propto T^4$, so $T \propto \left(r^{-2}\right)^{\frac{1}{4}}$, simplifying to $T \propto r^{-\frac{1}{2}}$.

Scale from the known ($T_{\text{Earth}} = 298\,\text{K}$, $r_{\text{Pluto}} = 40 r_{\text{Earth}}$):

$$T_{\text{Pluto}} \approx \frac{T_{\text{Earth}}}{\sqrt{40}} \approx \frac{298\,\text{K}}{6} \approx 50\,\text{K}$$

# What is Pluto's surface temperature?

Actual value: 44 K

[Source: `https://en.wikipedia.org/wiki/Pluto`]

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