On the photoelectric effect ANSWERS

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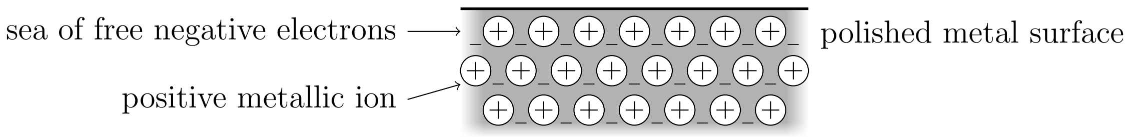
Warm-up problems

1. Draw a labelled diagram of the structure of a metal, and refer to it to explain two properties of metals.



This picture of metal structure explains:

- Metals conduct electricity (electrons can drift through structure under and applied electric field (p.d.))
- Metals are good conductors of heat (electrons carry energy through structure via collisions with ions)
- Metals are shiny (electrons react to incident light in such a way as to produce reflected light)
- Metals can release electrons when subjected to radiation (the electrons are loosely bound, and can be released if they gain energy)
- Metals are often malleable, ductile (sheets of ions can slide past each other, and still be held together by electrons)
- Metals are not brittle (crack formation is not encouraged by the metal structure)
- Metals usually have a high tensile strength (the electrostatic forces of attraction hold the structure together well)

2. Give the definition of 'a quantum', and explain how light is quantized.
A quantum is a packet of energy. Energy can only be absorbed / emitted in these quanta. Light can only travel along in packets of energy—quanta—called photons. These photons can behave like particles.

3. Write down an equation which gives the maximum kinetic energy of electrons escaping from a metal surface via the photoelectric effect, and explain each term carefully.

$$\frac{1}{2}m_e v^2 = hf - \phi,$$

where $\frac{1}{2}m_e v^2$ is the maximum kinetic energy of the escaping photoelectrons given in terms of their velocity v and the mass of an electron m_e; hf is the energy of the incoming photon which is related to its frequency f by Plack's constant h; and ϕ is the work function of the particular metal, which is a measure of how much energy is required to release an electron from its surface.

Regular problems

4. Calculate the energies of these photons.

   (a) A radio photon carrying a transmission of Test Match Special on BBC Radio 4 LW (which has a frequency of 198 kHz). What is its wavelength?

$$\begin{aligned} E &= hf \\ &= 6.63 \times 10^{-34}\,\mathrm{J}\,\cancel{\mathrm{s}} \times 198 \times 10^{3}\,\cancel{\mathrm{s}^{-1}} \\ &= 1.31 \times 10^{-28}\,\mathrm{J}. \end{aligned}$$

$$\begin{aligned} c &= f\lambda \\ \lambda &= \frac{c}{f} \\ &= \frac{3 \times 10^{8}\,\mathrm{m}\,\cancel{\mathrm{s}^{-1}}}{198 \times 10^{3}\,\cancel{\mathrm{s}^{-1}}} \\ &= 1.52\,\mathrm{km}. \end{aligned}$$

   (b) A visible photon emitted by the transition of an electron in a sodium atom (this is the brilliantly bright yellow light, familiar from street lamps, caused by the 'D-lines'[1] in the emission spectrum of sodium) at a wavelength of 589.0 nm.

$$\begin{aligned} E &= hf, \text{ but we can rewrite } f = \frac{c}{\lambda}, \text{ giving} \\ &= \frac{hc}{\lambda} \\ &= \frac{6.63 \times 10^{-34}\,\mathrm{J}\,\cancel{\mathrm{s}} \times 3 \times 10^{8}\,\cancel{\mathrm{m}\,\mathrm{s}^{-1}}}{589.0 \times 10^{-9}\,\cancel{\mathrm{m}}} \\ &= 3.38 \times 10^{-19}\,\mathrm{J}. \end{aligned}$$

   (c) A gamma photon detected in a PET (Positron Emission Tomography) scanner which has a wavelength of 2.43×10^{-12} m.

$$\begin{aligned} E &= \frac{hc}{\lambda} \\ &= \frac{6.63 \times 10^{-34}\,\mathrm{J}\,\cancel{\mathrm{s}} \times 3 \times 10^{8}\,\cancel{\mathrm{m}\,\mathrm{s}^{-1}}}{2.43 \times 10^{-12}\,\cancel{\mathrm{m}}} \\ &= 8.19 \times 10^{-14}\,\mathrm{J}. \end{aligned}$$

[1] There are actually two lines, very close together, at 589.0 nm and 589.6 nm, due to fine structure caused by the relativistic effect of spin-orbit coupling between L and S. This particular line, at 589.0 nm, is due to the transition from $3\mathrm{P}_{3/2} - 3\mathrm{S}_{1/2}$.

5. Explain the following

   (a) Electrons can be emitted from the surface of zinc when ultraviolet light falls on it, but not visible.
   The energy that a photon of light has is proportional to its frequency. Photoelectrons must get enough energy to escape from a **single** photon of light, and since UV light has a higher frequency than visible light, its photons will have enough energy to release photoelectrons, whereas visible photons will not. (In other words, the work function of zinc—the energy needed to release a photoelectron from its surface—must be between the energy of a visible and a UV photon.)

   (b) Electrons will be emitted from the surface of potassium even with visible light.
   The electrons in the surface of potassium require less energy to be freed (than those in zinc) and so both visible and UV photons are equally capable of releasing them. (It makes sense that the work function of potassium should be low as it is an alkali metal, in group 1 of the periodic table, and doesn't hold onto its outer electron very strongly.)

   (c) The electrons emitted from potassium have a greater kinetic energy than those from zinc do when they are illuminated with the same ultraviolet light.
   When a photoelectron is released from a metal surface, it absorbs the energy of the photon that releases it, and uses up some of this energy in escaping from the surface. Any left-over energy goes into its kinetic energy. Electrons escaping from potassium have to give up less energy to escape from the surface than electrons escaping from zinc do to leave a zinc surface. This means they will have more left-over, and therefore more kinetic energy, when they absorb the same kind of UV light.

6. Ultraviolet light of wavelength 3.5×10^{-7} m falls on a potassium surface. The work function of potassium is 4.4×10^{-19} J.

   (a) What is the maximum energy of the photoelectrons?

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\begin{aligned}
E &= \frac{hc}{\lambda} - \phi \\
&= \frac{6.63 \times 10^{-34}\,\mathrm{J\cancel{s}} \times 3 \times 10^{8}\,\cancel{\mathrm{m s^{-1}}}}{3.5 \times 10^{-7}\,\cancel{\mathrm{m}}} - 4.4 \times 10^{-19}\,\mathrm{J} \\
&= 1.28 \times 10^{-19}\,\mathrm{J}.
\end{aligned}
$$

   (b) Why is this a maximum energy (i.e. why might electrons be emitted with less than this energy)?
   The work function ϕ is the minimum amount of energy which is required to remove an electron from the surface. The electrons which are emitted might come from e.g. deeper inside the surface, in which case they would need more energy to release them, leaving them with less energy from the incoming photon left-over (as kinetic energy).

7. X-rays with frequency 1.53×10^{16} Hz cause the emission of electrons from a material with a maximum kinetic energy of 2.18×10^{-18} J. Calculate, for this material,

(a) the work function,

$$\begin{aligned}\phi &= hf - E_k \\ &= 6.63 \times 10^{-34}\,\text{J}\cancel{\text{s}} \times 1.53 \times 10^{16}\,\cancel{\text{s}^{-1}} - 2.18 \times 10^{-18}\,\text{J} \\ &= 7.96 \times 10^{-18}\,\text{J}.\end{aligned}$$

(b) the threshold frequency.

$$\begin{aligned}hf &= \phi \\ f &= \frac{\phi}{h} \\ &= \frac{7.96 \times 10^{-18}\,\cancel{\text{J}}}{6.63 \times 10^{-34}\,\cancel{\text{J}}\text{s}} \\ &= 1.20 \times 10^{16}\,\text{Hz}.\end{aligned}$$

8. [AQA PA01 Jan 2002]

(a) When monochromatic light is incident on a metal plate, electrons are emitted only when the frequency of the light exceeds a certain threshold frequency.

i. Explain, in terms of energy, why this threshold frequency exists. [3]

- the energy of each photon (or 'the light') increases with frequency
- electrons need a minimum amount of energy to leave the **metal**
- this amount of energy is equal to the work function

(b) A sodium metal surface is illuminated with incident light of frequency 9.70×10^{14} Hz. The maximum kinetic energy of an emitted electron is 2.49×10^{-19} J.
Calculate

i. the wavelength of the incident light,

$$\begin{aligned}(\text{use of } v = f\lambda \text{ gives})\ \lambda &= \frac{3.00 \times 10^{8}}{9.70 \times 10^{14}} \\ &= 3.09 \times 10^{-7}\,\text{m}\end{aligned}$$

ii. the energy, in J, of each incident photon,

$$\begin{aligned}(\text{use of } E = hf \text{ gives})\ E &= 6.63 \times 10^{-34} \times 9.70 \times 10^{14} \\ &= 6.43 \times 10^{-19}\,(\text{J})\end{aligned}$$

iii. the work function, in J, of sodium,

$$\begin{aligned}(\text{use of } hf = \phi + E_k \text{ gives})\ 6.43 \times 10^{-19} &= \phi + 2.49 \times 10^{-19} \\ \phi &= 3.94 \times 10^{-19}\,(\text{J})\end{aligned}$$

iv. the work function, in eV, of sodium. [7]

$$\begin{aligned}\phi &= \left(\frac{3.94 \times 10^{-19}}{1.60 \times 10^{-19}}\right) \\ &= 2.46\,(\text{eV})\end{aligned}$$

Extension problems

8. The maximum kinetic energy of photoelectrons from aluminium is found experimentally to be 2.3 eV for radiation of 2.00×10^{-7} m and 0.90 eV for radiation of 3.13×10^{-7} m. Use these experimental data to calculate Planck's constant and the work function for the metal.
   Hint: EITHER use a simultaneous equation method, with two equations of form $E_k = \frac{hc}{\lambda} - \phi$, with h and ϕ being the two unknowns; OR convert λ to f, plot a graph of E_k against f and realize that, since $E_k = hf - \phi$, this will be a straight line of gradient h and y-intercept $-\phi$ (or x-intercept $\frac{\phi}{h}$).

9. (a) If the energy flux associated with a beam of light of wavelength 3×10^{-7} m is $10\,\mathrm{W\,m^{-2}}$, estimate how long it would take, classically, for sufficient energy to arrive at a potassium atom of radius 2×10^{-10} m so than an electron can be ejected. The work function of potassium is 4.4×10^{-19} J.
   (b) What would be the average emission rate of photoelectrons if such light fell on a piece of potassium $10^{-3}\,\mathrm{m^2}$ in area?
   (c) Would you expect the average emission rate to be significantly affected by quantum mechanical considerations?