On uncertainty

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Questions 2, 3, 4, 9 and 10 are from *Measurements and their Uncertainties*, Hughes & Hase. Questions 5–8 are from *Upgrade your physics*, Machacek.

Warm-up problems

1. Explain the difference between *absolute uncertainty* and *percentage undertainty.*

   The absolute uncertainty is the range of values on either side of a measurements within which the true value is expected to be located (at least, with a certain probability). Absolute uncertainties have the same units as the quantity being measured.

   An expected/probable difference between the result of any measurement and the true value of the quantity being measured can also be reported as a percentage of the value, e.g. 5%. Percentage uncertainties have no units.

2. Round the following numbers to (a) two significant figures and (b) four significant figures.

   (i) 602.20
   (a) 600 (b) 602.2

   (ii) 0.001 380 6
   (a) 0.0014 (b) 0.001 381

   (iii) 0.022 413 83
   (a) 0.022 (b) 0.022 41

   (iv) 1.602 19
   (a) 1.6 (b) 1.6022

   (v) 91.095
   (a) 91 (b) 91.10

   (vi) 0.1660
   (a) 0.17 (b) 0.1660

   (vii) 299 790 000
   (a) 300 000 000 (b) 299 800 000

   (viii) 66.2617
   (a) 66 (b) 66.26

   (ix) 0.000 000 667 2
   (a) 0.000 000 67 (b) 0.000 000 667 2

   (x) 3.141 593
   (a) 3.1 (b) 3.142

3. Rewrite the ten numbers from question 2 in scientific notation.

   (i) 6.02×10^2
   (ii) 1.38×10^{-3}
   (iii) 2.24×10^{-2}
   (iv) 1.60×10^0
   (v) 9.11×10^1
   (vi) 1.66×10^{-1}
   (vii) 3.00×10^8
   (viii) 6.63×10^1
   (ix) 6.67×10^7
   (x) 3.14×10^0

Regular problems

4. A car covers a distance of 250 m in 13 s; the average speed is calculated to the 10 decimal places of the calculator as $19.230\,769\,23\,\mathrm{m\,s^{-1}}$. Explain why it is incorrect to believe all of the significant figures of the quoted speed.

   The distance is given as 250 m. This suggests that it is known to the nearest 10 m or 1 m at best,[1] an percentage uncertainty of 0.4%.

   The time is given as 13 s, implying that it is known to the nearest second, a % uncertainty of 8%.

   This means the final answer can at best be known to the nearest 2 m, so following the conventions, the best answer would be $(19 \pm 2)\,\mathrm{m\,s^{-1}}$.

   The rule of thumb to be followed (without going through a full uncertainty calculation for every question) is: give the final answer to the same number of significant figures as the least number of s.f. in the values given in the question. In this case, that would be 2 s.f., so the final answer should be quoted as $19\,\mathrm{m\,s^{-1}}$.

5. Work out the percentage uncertainty when a 5 V battery is measured to the nearest 0.2 V.

   In this case the absolute uncertainty is ± 0.1 V, and this makes the percentage uncertainty

$$\frac{0.1\,\mathrm{V}}{5\,\mathrm{V}} \times 100\% = 2\%.$$

6. If I don't want to have to correct my watch more than once a week, and I never want my watch to be more than 1s from the correct time, calculate the necessary maximum relative uncertainty of the electronic oscillator which I can tolerate.

   You can only tolerate an absolute uncertainty of ± 1 s in one week. As a percentage uncertainty, this is

$$\frac{1\,\mathrm{s}}{1\,\mathrm{week} \times \frac{7\,\mathrm{day}}{1\,\mathrm{week}} \times \frac{24\,\mathrm{h}}{1\,\mathrm{day}} \times \frac{60\,\mathrm{min}}{1\,\mathrm{h}} \times \frac{60\,\mathrm{s}}{1\,\mathrm{min}}} \times 100\% = \pm 2 \times 10^{-4}\%.$$

[1] Standard form would be better here, as 2.50×10^2 m would imply ± 1 m and 2.5×10^2 m would imply ± 10 m, making thing much clearer.

This level of accuracy in timekeeping has been consistently achieved only in recent times (with the invention of pressure compensation for pendulum clocks in the 1830s). Modern quartz crystal watches also come close to this level of accuracy (a cheap quartz wristwatch comes in at around $\pm 2 \times 10^{-4}\%$ or 1 second per day).

7. My two-storey house is $(7.05 \pm 0.02)\,\mathrm{m}$ tall. The ground floor is $(3.20 \pm 0.01)\,\mathrm{m}$ tall. How tall is the first floor?

$$7.05\,\mathrm{m} - 3.2\,\mathrm{m} = 3.85\,\mathrm{m}$$

To find the uncertainty, add the absolute uncertainties, giving $0.02\,\mathrm{m} + 0.01\,\mathrm{m} = \pm 0.03\,\mathrm{m}$. The final answer is thus $(3.85 \pm 0.03)\,\mathrm{m}$.

8. I want to measure the resistance of a resistor. My voltmeter can read up to 5 V, with an absolute uncertainty of $\pm 0.1\,\mathrm{V}$. My ammeter can read up to 1 A with an absolute uncertainty of 0.02 A. Assuming that my resistor is approximately $10\,\Omega$, calculate the absolute uncertainty of the resistance I measure using the formula $R = V/I$. Assume that I choose the current to make the relative uncertainty as small as possible.

To minimize the relative uncertainty, we need to make the readings as large as possible. Using a voltage of around 5 V will give a current $I = \frac{V}{R} = 0.5\,\mathrm{A}$, so the uncertainty in voltage will be $\frac{0.1\,\mathrm{V}}{5\,\mathrm{V}} \times 100\% = 2\%$ and the uncertainty in current will be $\frac{0.02\,\mathrm{A}}{0.5\,\mathrm{A}} \times 100\% = 4\%$. The final uncertainty is $2\% + 4\% = 6\%$, which will be about $\pm 0.6\,\Omega$.

9. Fifteen measurements of a resistance are quoted here, based on approximately 10 repeat measurements. Only three of them are reported correctly. Identify the mistakes in the other results.

   (i) $(99.8 \pm 0.270) \times 10^3\ \Omega$
   Too many s.f. in uncertainty; value not given to same d.p. as uncertainty. Should read $(99.8 \pm 0.3) \times 10^3\,\Omega$.

   (ii) $(100 \pm 0.3) \times 10^3\ \Omega$
   Value not given to same d.p. as uncertainty. Should read $(100.0 \pm 0.3) \times 10^3\,\Omega$.

   (iii) $(100.0 \pm 0.3) \times 10^3\,\Omega$

   (iv) $(100.1 \pm 0.3) \times 10^3$
   No unit.

   (v) $97.1 \times 10^3 \pm 276\ \Omega$
   Too many s.f. in uncertainty; value not given to same d.p. as uncertainty. Should read $(97.1 \pm 0.3) \times 10^3\,\Omega$.

   (vi) $(99.8645 \pm 0.2701) \times 10^3\ \Omega$
   Too many s.f. in uncertainty; value not given to same d.p. as uncertainty. Should read $(99.8 \pm 0.3) \times 10^3\,\Omega$.

   (vii) $98.6 \times 10^3 \pm 3 \times 10^2\,\Omega$
   Value not given to same d.p. as uncertainty. Should read $(98.6 \pm 0.3) \times 10^3\,\Omega$.

   (viii) $99.4 \times 10^3 \pm 36.0 \times 10^2\,\Omega$
   Too many s.f. in uncertainty; value not given to same d.p. as uncertainty. Should read $(99 \pm 4) \times 10^3\,\Omega$.

(ix) $101.5 \times 10^3 \pm 0.3 \times 10^1\,\Omega$
Value not given to same d.p. as uncertainty. Should read $(101\,500 \pm 3)\,\Omega$.

(x) $(99.8 \pm 0.3) \times 10^3\,\Omega$

(xi) $95.2 \times 10^3 \pm 273\,\Omega$
Too many s.f. in uncertainty; value not given to same d.p. as uncertainty. Should read $(95.2 \pm 0.3) \times 10^3\,\Omega$.

(xii) $98,714 \pm 378\ \Omega$
Too many s.f. in uncertainty. Should read $(98\,700 \pm 400)\,\Omega$.

(xiii) $99000 \pm 278\ \Omega$
Too many s.f. in uncertainty. Should read $(99\,000 \pm 300)\,\Omega$.

(xiv) $98,714 \pm 3 \times 10^3\ \Omega$
Value not given to same d.p. as uncertainty. Should read $(99\,000 \pm 3000)\,\Omega$ or $(99 \pm 3) \times 10^3\,\Omega$.

(xv) $(98\,900 \pm 300)\,\Omega$

Extension problems

10. The angle of refraction θ_r for a light ray in a medium of refractive index n which is incident from a vacuum at an angle θ_i is obtained from Snell's law: $n \sin\theta_r = \sin\theta_i$. Calculate θ_r and its associated error if $\theta_i = (25.0 \pm 0.1)^\circ$ and n=1.54 ± 0.01.

    For complicated functions like this, first make a calculation of the quantity required in the usual way:

$$\theta_r = \left(\frac{\sin\theta_i}{n}\right) = 15.9^\circ.$$

    Note that the final angle is unlikely to come out better than the input angle, i.e. $\pm 0.1^\circ$. Now check what the significance of the uncertainty in θ_i is by checking the difference between $\sin 25.0^\circ$ and $\sin 25.1^\circ$:

$$\frac{\sin 25.1^\circ - \sin 25.0^\circ}{\sin 25.1^\circ} \times 100\% = \frac{0.4242 - 0.4226}{0.4242} \times 100\% = 0.4\%$$

    This can then be combined with the uncertainty in refractive index ($\frac{0.01}{1.54} = 0.6\%$) in the usual way, giving a total uncertainty of 1%. The final result is therefore quoted as

$$(15.9 \pm 0.2)^\circ.$$