Nuclear mass and energy

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Mass and energy

The atomic mass unit, u

When measuring the masses of atoms, it is not convenient to use the kilogram, as it is too large a unit. A smaller unit, the atomic mass unit, u, is used. This is defined as

$$1\,\mathrm{u} = \frac{1}{12}\text{mass of carbon-12 atom}$$

Experimentally, it can be shown that one atomic mass unit is equal to $1.660\,538\,682 \times 10^{-27}$ kg. This is, of course, very close to the mass of a nucleon (i.e. a proton or a neutron). Therefore, atoms tend to have atomic masses close to whole numbers.

	m/u
Electron	0.000549
Neutron	1.008665
Proton	1.007276
Hydrogen atom (p+e)	1.007825
Helium atom (2p+2n+2e)	4.002603
Alpha particle (2p+2n)	4.001505
Carbon-12 atom (6p+6n+6e)	12.000000

NB This is a rare occasion in physics where the numbers used are given to large number of significant figures.

Note that it is not simply a case of adding up masses of the protons and neutrons and electrons, to get the atomic mass. The atomic mass of an isotope will always be less than the sum of its constituent parts. The above values have to be measured experimentally.

It is important to distinguish between atomic mass and nuclear mass. Atomic mass (which is usually the one given) includes the mass of all the electrons, whereas nuclear mass only includes the nucleons. The nuclear mass can simply be found by taking the correct number of electrons off the atomic mass.

Energy equivalence of mass

Special relativity tells us that energy and mass are equivalent, and can be interchanged. They are related by

$$E = mc^2$$

where E is the energy in joules, m is the mass in kilograms and c is the speed of light ($3 \times 10^8\,\mathrm{m\,s^{-1}}$.

Einstein said that "if a body gives off energy E in the form of radiation, its mass diminishes by E/c^2". This relationship is mostly often used in nuclear physics, but applies throughout the whole of physics. If a hot object cools, as well as a temperature fall, the mass decreases.

This relationship allows us to calculate the interchange between mass and energy. e.g. 1kg of mass has an energy equivalence of 9 0e16J, and 1 u of mass has an energy equivalence of 1.49×10^{-10} J.

To allow more convenient change between mass and energy, we need to introduce a different energy unit to the joule. We can use the electron-volt (eV), which is defined as

$$1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}.$$

Therefore, as $1\,\mathrm{u} = 1.49 \times 10^{-10}\,\mathrm{J}$, so $1\,\mathrm{u} = \frac{1.4 \times 10^{-10}}{1.6 \times 10^{-19}}\mathrm{eV} = 9.31 \times 10^8\,\mathrm{eV}$, which is more commonly written as

$$1\,\mathrm{u} = 931.3\,\mathrm{MeV}.$$

This is a very useful relationship which allows us to convert between masses (in u) and energies (in MeV) directly, without having to apply $E = mc^2$. If masses are given in kg, and energies in J, then it is necessary to convert using $E = mc^2$.

Mass difference

If the mass of a nucleus is measured, it turns out to be *less* than the mass of the individual constituents that make up the nucleus. The difference between the mass of the nucleus and its components is called the *mass difference* (or *mass defect*).

$$\text{mass difference} = \text{mass of individual nucleons} - \text{mass of nucleus}$$

The mass difference is most commonly measured in atomic mass units, u.

Example Calculate the mass difference of an alpha particle.

$$\begin{aligned}\text{mass difference} &= \text{mass of individual nucleons} - \text{mass of nucleus}\\ &= [(2 \times 1.007\,276\,\text{u}) + (2 \times 1.008\,665\,\text{u})] - 4.001\,505\,\text{u}\\ &= 4.031\,882\,\text{u} - 4.001\,505\,\text{u}\\ &= 0.303\,77\,\text{u}\end{aligned}$$

The mass difference arises as when the individual nucleons are pushed together to form the nucleus, gamma rays are released, which causes a decrease in energy, and therefore a decrease in mass. The energy of the gamma rays is equal to the mass difference.

The mass difference is different for different nuclei, and even for different isotopes. It is not possible to predict what the mass of any individual nucleus is; they have to be experimentally measured.