Elasticity

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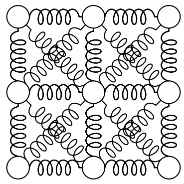
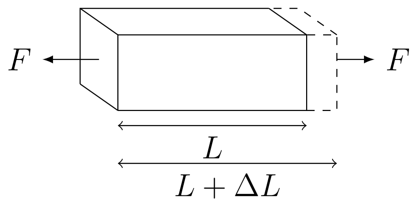
Elasticity of materials

All materials are elastic to some extent, due to the electrostatic nature of the forces that hold their particles together. We can imagine the bonds in between the particles as tiny springs[1], i.e. Hooke's law applies.



When you push on a piece of material, it 'gives', and we find that so long as the distortion of the material is not too great—otherwise it will break apart or be crushed—the force applied to the body will be proportional to the distortion. The amount of force for which this is true, and the material behaves elastically, depends on the material: for dough or blu-tack it is very small, but for steel it is relatively large.



A block of material of length L will thus increase temporarily in length by an amount ΔL whilst a tensile force[2] is applied, as shown above. This change in length will be a small fraction of the original length (indeed, materials like wood and steel will break if the change in length is more than

[1] To fully understand such intermolecular forces requires quantum physics—they are due to the vastly complicated interactions of the electrons and nuclei in one molecule with those in its neighbours—but it turns out that for very small displacements, the force is proportional to the displacement to a very good approximation.

[2] *Tensile* means 'stretching', and for some materials, e.g. rubber, a *compressive* force is important and different.

a few percent of the original length). We already know that in this limit of small extensions—i.e. up to the elastic limit—the material will obey

$$F \propto \Delta L.$$

Stress and Strain

The lengthening ΔL of the bar will depend on its length: if two blocks were joined together end-to-end the same forces would act on each, so they would each stretch by ΔL. The stretch in a block of length $2L$ would be double that of one half as long. To get a measure of the deformation which is more a property of the material, rather than a particular shape, we introduce a new quantity called *tensile strain*, usually denoted by ε, which is the ratio $\Delta L/L$ of the extension to the original length. It has no unit, as it is a dimensionless ratio. This is proportional to the force, but independent of the length L of the material:

$$F \propto \frac{\Delta L}{L}.$$

The force F needed to extend the bar by a given extension ΔL depends on the cross sectional area A of the block. If two blocks are placed side by side, a force F on each of these is needed, or $2F$ altogether, to extend them both by ΔL. The force, for a given amount of stretch, must be proportional to the area, so we choose to introduce a quantity called *tensile stress*, usually denoted by σ, which is the tensile force per cross sectional area. It therefore has the SI unit of $\mathrm{N\,m^{-2}}$ or Pa. Thus we can express Hooke's law where the constant of proportionality does not depend on the shape or size of the block, and is a property of the material:

$$\frac{F}{A} = Y\frac{\Delta L}{L},$$

$$\sigma = Y\varepsilon, \text{ or}$$

$$\text{stress} = \text{Young modulus} \times \text{strain}$$

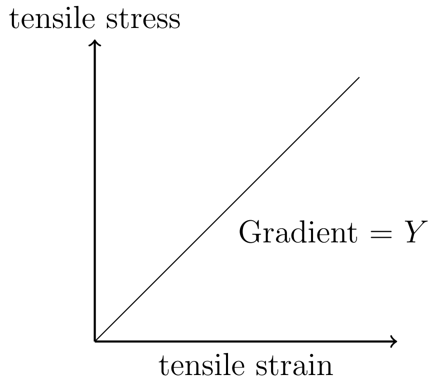
The Young modulus

The constant of proportionality between stress and strain, which is independent of the dimensions of the material, is called the Young modulus of elasticity, Y.[3] Its SI unit is the Pa, but it is normally expressed in GPa.

[3] It is denoted by E in some books, but as E is already overloaded in physics (energy, electric field strength, emf), I prefer Y instead.

Material	Y/GPa
Nylon	2–4
Human bone	14
Aluminium	69
Glass	50–90
Steel	210
Diamond	1220

A stress–strain graph will have the same shape as a force–extension graph, and the Young modulus proportionality (just a version of Hooke's law) applies to the straight line section of this graph. The gradient of this line will give the Young modulus.



Measuring Y

The Young modulus is commonly measured for a wire by using a wire with a vernier scale with a variable load hanging from it next to a wire with a mm scale with a fixed load to keep it taut. As the load is varied, the vernier scale will move allowing the extension to be measured.

The length of the wire and its cross sectional area must be determined, allowing a graph of stress against strain to be plotted. The main mm scale must be attached to the same support as the test wire, so that any sagging of the support or changes due to temperature will not be included in the measurements.