Elasticity

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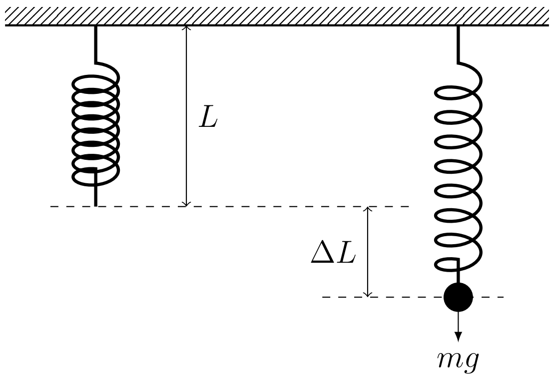
Hooke's law

Elasticity is the behaviour of materials which will recover their original size and shape when forces which deform them are removed. We start by discussing springs, which are formed by winding a wire round and round along a uniform cylinder (thus forming a helix; such springs are sometimes called 'helical springs').

A spring can support a weight hanging from it, and in doing to it extends by a certain amount. The load which hangs from it is pulled towards the ground by its weight force, and this produces a tiny twist thoughout the length of the wire, which results in a small vertical deflection in each turn (adding up to a large deflection if there are many turns). The spring has to provide an force of tension equal to the weight of the load to keep it supported.

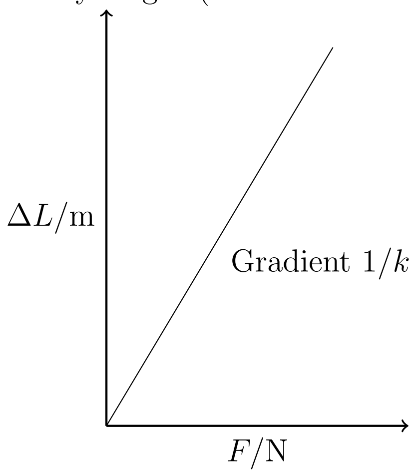


The way in which the length of a spring depends on the force on it was first investigated in the seventeenth century by Robert Hooke. He discovered that *the extension of a spring from its natural length is directly proportional to the force applied to it* and this is known as Hooke's law. Mathematically,

$$F \propto \Delta L, \text{ or}$$

$$F = k\Delta L,$$

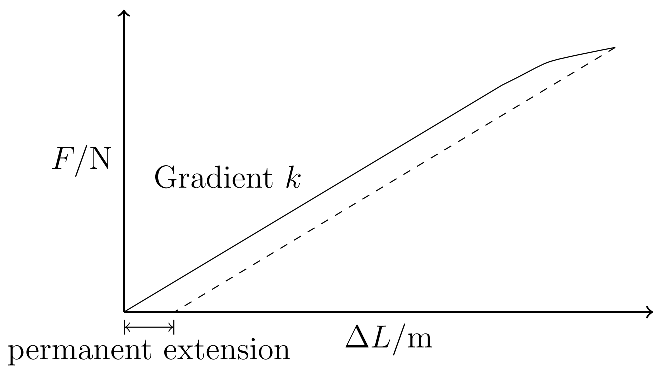
where F is the force applied to the spring, ΔL is the (stretched) length of the spring, L is the original length of the spring. The constant of proportionality k is called the *spring constant* (or stiffness constant), and it will have the dimensions of force divided by length (which is $\mathrm{N\,m^{-1}}$ in the SI system).



e.g. A typical spring which you might investigate in school will be stretched by about 5 cm by the weight of a 100 g mass. The weight force applied in this case is 1 N, and so the spring constant of such a spring would be $20\,\mathrm{N\,m^{1}}$. This could be checked by an experimental investigation measuring the extensions produced by different loads. The gradient of the graph of load against extension is the spring constant k (or the gradient of extension against load is $1/k$).

Elastic limit

A spring will not go on obeying Hooke's law indefinitely: if it continues to be stretched, there will be a value of force where the extension produced stops being proportional to the load applied. If forces above this *elastic limit* are applied to the spring, the spring will be permanently stretched once the forces are removed.



NB The axes on this graph are the other way around

The graph shows a spring which has been stretched beyond its elastic limit—the extension is no longer proportional to the force applied—and if the force were now to be removed, the spring would follow the dotted path, leaving it permanently deformed; it will no longer return to its natural length.

Energy stored in a stretched spring

Usually springs are used in mechanical devices, and to design these it is often necessary not only to calculate how much force will be required to stretch or compress a spring by a certain amount, but also how much energy this will take. This will equal the amount of energy stored up as elastic potential energy in the stretched or compressed spring.

During the process of stretching a spring, the work done is given by

$$\text{Work done} = \text{Force} \times \text{distance}.$$

This equals the area between the load–extension graph and the extension axis (not necessarily the 'area under the curve' if the axes are the wrong way round and the graph is a curve—e.g. if the elastic limit has been passed—as the areas between the curve and the two axes will be different).

For the straight line section of the graph, before the elastic limit, where Hooke's law applies, this area between the line and the extension force is the average force multiplied by the extension:

$$\begin{aligned}
\text{energy stored in spring} &= \text{work done in stretching the spring} \\
&= \text{average force} \times \text{extension} \\
&= \frac{1}{2}\text{maximum force} \times \text{extension} \\
&= (\frac{1}{2}k\Delta L) \times \Delta L \\
&= \frac{1}{2}k(\Delta L)^2.
\end{aligned}$$