On density

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Warm-up problems

1. Write down the equation used to calculate density, two units density can be measured in, and show how to convert between them.
2. Put the following substances in order of increasing density: water, xenon, iron, cork, olive oil, air, uranium.
3. Write down three contexts in which measurements of density can be useful from class.

Regular problems

4. From the 'density' exercise in your homework booklet:
   (a) Answer question 1.
   (b) Answer question 2.
5. Water starts to fill a paddling pool by flowing out of a hosepipe of inner diameter 2 cm at a rate of $1.5\,\mathrm{m\ s^{-1}}$. What mass of water is there in the paddling pool after 1 minute? ($\rho_{\mathrm{water}} = 1000\,\mathrm{kg\ m^{-3}}$).
6. Answer question 3 from the 'density' exercise in your homework booklet.
7. Explain why solid objects made from some substances float on water and some sink:
   (a) using the idea of density to give a general rule of thumb for substances,
   (b) using the idea of upthrust force (Archimedes' principle) on an object,
   (c) using the idea of pressure forces on an object.

Extension problems

8. The average wind speed at 25.0 m above the Earth's surface, on top of a hill, is $7.0\,\mathrm{m\,s^{-1}}$. A wind turbine on the top of a hill has the rotational centre of its blades at 25.0 m above the ground and has blades of length 5 m. The density of the air is $1.3\,\mathrm{kg\,m^{-3}}$.
   (a) Imagine the circle described by the tips of the rotating blades. What is the volume of air passing through this circle every second?
   (b) What is the mass of this air?

(c) What is the kinetic energy of this mass of air?

(d) If the turbine converts 40% of the wind energy into electrical energy, what is the power output of the generator?

[British Physics Olympiad Physics Challenge 2004 q.14, cf. AQA PHYA2 Jan 2010 q.7]

9. A votive crown made by command of king Hierto II of Syracuse was suspected of having been fraudulently made in part from silver substituted in place of the gold supplied by the king. Archimedes realized that by comparing the density of the crown to the densities of gold and silver, which are $10.50\,\mathrm{g\,cm^{-3}}$ and $18.90\,\mathrm{g\,cm^{-3}}$ respectively, he could calculate the composition of the crown.[1] If Archimedes found that the crown weighed 3.52 kg in air and 3.31 g in water, find the composition, by mass, of the alloy, assuming that there has been no volume change in the process of producing the alloy.

[Duncan, Physics: *A textbook for advanced level students*, p.205 q.3, adapted]

[1]This story is told by Vitruvius, a roman architect in the first century BC, who tells us that Archimedes got the idea from taking a bath which overflowed when he immmersed his body. He thereupon is reported to have leapt out of the bath in joy, and, returning home naked, cried out with a loud voice that he had found what he was searching for, exclaiming, in Greek $\varepsilon\upsilon\rho\eta\kappa\alpha$, (I have found it out). [Vitruvius, *De Architectura*, Book IX, paragraphs 9–12] It is unlikely that Archimedes in fact used this method. Galileo pointed out that it is far more likely that Archimedes in fact used some form of balance to determine the composition of the crown, making use of his law of bouyancy and his law of the lever. [Galileo Galilei, *La Bilancetta*, published in *Opera di Galileo Galilei*, ed. Franz Brunetti, 1980]