Universal Gravitation

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Warm up problems

1. Calculate the gravitational pull of the Earth on the Moon, if the mass of the moon is 7.4×10^{22} kg, and the mass of the Earth is 6.0×10^{24} kg, and the distance between their centres is 3.8×10^{8} m.
2. Explorer 38 is a radio-astronomy research satellite of mass 200 kg, that orbits at a height above the Earths surface equal to half the Earth's radius. Given that the radius of the Earth is 6.4×10^{6} m, and the mass of the Earth is 6.0×10^{24} kg, calculate the gravitational force on the satellite.
3. The weight of a 10 kg mass on the Earth's surface is approximately 98 N, and the radius of the Earth is 6.4×10^{6} m. Use this information to calculate a value for the mass of the Earth.

4 Two lead spheres of radius 50 mm just touch each other. Calculate

(a) the volume of the spheres, in m^3,
(b) the mass of the spheres, if the density of lead is 11000 kg m^{-3},
(c) the gravitational force of attraction between them.

5 If a satellite was placed on the surface of a planet of radius r, it would experience a force of F. Show that if it were put in an orbit at a height of $r/50$ above the planet's surface, the force on the planet would be $0.96F$.

Lesson Objectives

1. To use Newton's law to do some questions.
2. To have a look at the *universality* of Newton's law.
3. To look at circular orbits

REMINDER: Office hours are week **1** Tuesdays 3.45–5.0 p.m. in room 19.
Next office hours: Tuesday 11 September 2012

Specification Requirement

Orbits of planets and satellites
Orbital period and speed related to radius of circular orbit.

[AQA GCE AS and A Level Specification Physics A, 2009/10 onwards]

Kepler's laws

I. Each planet moves around the Sun in an ellipse, with the Sun at one focus
II. The line from the Sun to the planet sweeps out equal areas in equal times
III. The squares of the orbital periods of the planets are proportional to the cubes of the semi-major axes of their orbits: $T^2 \sim a^3$

Aim: To prove Kepler's 3rd law *for a circular orbit*, that the square of the orbital period T^2 is proportional to the cube of the radius r^3 of the orbit,

$$T^2 = r^3.$$

Starting point: what you already know about gravity $F = G\frac{m_1 m_2}{r^2}$, and what you already know about circular motion...
You have 5 mins!

Hint

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = \frac{mv^2}{r} = mr\omega^2$$

Then you need $v(T, r)$ or $\omega(T, r)$

Proof

$$F = \frac{GMm}{r^2}, \quad F = \frac{mv^2}{r}$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

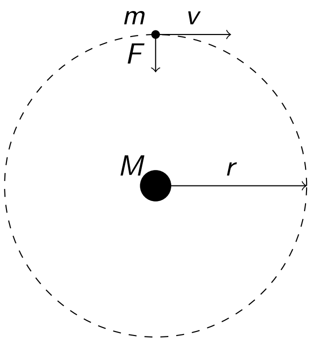
$$\frac{GM}{r} = v^2$$

For motion in a circle, $v = \frac{2\pi r}{T}$

$$\frac{GM}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$T^2 = \frac{4\pi^2}{GM} r^3$$



Having confidence in Newton. . .

Corrections to Newton. . .

Newton's laws (like any current physical theory!) is only an approximation to the real world. Einstein published the general theory of relativity in 1915, which includes the Einstein Field Equations (EFE):

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

These correct Newton's laws, and make experimental predictions that Newton's law of universal gravitation doesn't, e.g. deflection of light by gravitational fields, the curvature of space–time, the big bang, black holes, the precession of Venus' orbit.
Newton's law can be seen as a special case of the equation above, where gravity fields are weak, and all speeds are much less than the speed of light.