Newton's Law of Gravitation

A.C. Norman

Bishop Heber High School

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Lesson Objectives

1. To revise circular motion calculations.
2. To learn Newton's law of universal gravitation.
3. To do some calculations based on Newton's law.

REMINDER: Office hours are week **1** Tuesdays 3.45–5.0 p.m. in room 19.
Next office hours: Tuesday 11 September 2012

Specification Requirement

Newton's law

Gravity as a universal attractive force acting between all matter.
Force between point masses

$$F = \frac{Gm_1 m_2}{r^2},$$

where G is the gravitational constant.

[AQA GCE AS and A Level Specification Physics A, 2009/10 onwards]

1 A particle moves along a circular path of radius 3.0 m with an angular velocity of 20 rad s^{-1}. Calculate:

(a) the linear speed of the particle
(b) the angular velocity in revolutions per second
(c) the time for one revolution
(d) the centripetal acceleration.

2. A particle of mass 0.2 kg moves in a circular path with an angular velocity of 5 rad s^{-1} under the action of a centripetal force of 4 N. What is the radius of the path?
3. What force is required to cause a body of mass 3.0 g to move in a horizontal circle of radius 2 m at a constant rate of 4 revolutions per second?

4 A particle of mass 80 g rests 16 cm from the centre of a turntable. If the maximum frictional force between the particle and the turntable is 0.72 N, what is the maximum angular velocity at which the turntable could rotate without the particle slipping?

The Theory of Gravitation

every object in the universe attracts every other object with a force which for any two bodies varies inversely as the square of the distance between them

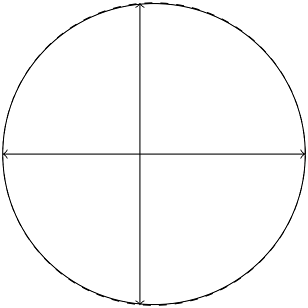
$$F = G\frac{m_1 m_2}{r^2}$$

History

- AD 150 Ptolemy: Earth at centre of solar system, with Sun rotating around it, and planetary orbits describes by a combination of circles (epicycles)
- Tycho Brahe (1546–1601) made observations of planetary and stellar positions to an accuracy of 10 arcsec (the resolution of the human eye is 1 arcmin), and made voluminous tables
- Johannes Kepler (1571–1630) spent 5 year fitting circles to Tycho's data for Mars' orbit and found difference of the order of 8 arcmin. Rejected the model due to the known accuracy of Tycho's data.
- Eventually Kepler concluded that the orbits were ellipses.

Mars data

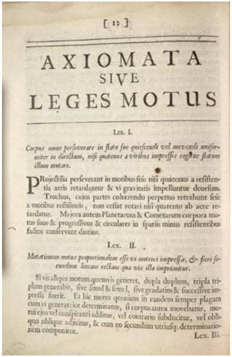
- The solid line is an ellipse $r(\theta) = \frac{l}{1+\epsilon\cos\theta}$ of eccentricity $\epsilon = 0.1$
  [This is the eccentricity of Mars' orbit]
- The dotted line shows a circle fitted to the ellipse
- Tycho's date and Kepler's data-fitting were good enough that Kepler was able to reject the circle hypothesis!



Kepler's laws

I. Each planet moves around the Sun in an ellipse, with the Sun at one focus
II. The line from the Sun to the planet sweeps out equal areas in equal times
III. The squares of the orbital periods of the planets are proportional to the cubes of the semi-major axes of their orbits: $T^2 \sim a^3$

Philosophiæ Naturalis Principia Mathematica,
Isaac Newton, 1687



[12]

AXIOMATA
SIVE
LEGES MOTUS

Lex. I.

Corpus omne perſeverare in ſtatu ſuo quieſcendi vel movendi uniformiter in directum, niſi quatenus a viribus impreſſis cogitur ſtatum illum mutare.

Projectilia perſeverant in motibus ſuis niſi quatenus a reſiſtentia aeris retardantur & vi gravitatis impelluntur deorſum. Trochus, cujus partes cohærendo perpetuo retrahunt ſeſe a motibus rectilineis, non ceſſat rotari niſi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus ſuos & progreſſivos & circulares in ſpatiis minus reſiſtentibus factos conſervant diutius.

Lex. II.

Mutationem motus proportionalem eſſe vi motrici impreſſæ, & fieri ſecundum lineam rectam qua vis illa imprimitur.

Si vis aliqua motum quemvis generet, dupla duplum, tripla triplum generabit, ſive ſimul & ſemel, ſive gradatim & ſucceſſive impreſſa fuerit. Et hic motus quoniam in eandem ſemper plagam cum vi generatrice determinatur, ſi corpus antea movebatur, motui ejus vel conſpiranti additur, vel contrario ſubducitur, vel oblique oblique adjicitur, & cum eo ſecundum utriuſq; determinationem componitur.

Lex. III.

Newton introduced the idea of *forces*, and from his better understanding of the theory of motion, he could appreciated that the force needed to make Kepler's laws true would be *towards the Sun*.

He was a man of considerable feeling for generalities, and supposed that the force holding planets around the Sun was the same as that holding Jupiter's moons, and us on the Earth.

He proposed a universal force—that *everything pulls everything else*!

Moons of Jupiter

Questions

1 Express Newton's law of gravitation in symbols.

2 Show how the units of G can be expressed as $m^3\ kg^{-1}\ s^{-2}$.

3 (a) Calculate the gravitational force of attraction between two 0.5 kg masses which are 2 m apart.

(b) Use the previous answer to calculate the force if the masses were

(i) 4 m apart

(ii) 6 m apart

Questions

4. Calculate the gravitational pull of the Earth on the Moon, if the mass of the moon is 7.4×10^{22} kg, and the mass of the Earth is 6.0×10^{24} kg, and the distance between their centres is 3.8×10^{8} m.
5. Explorer 38 is a radio-astronomy research satellite of mass 200 kg, that orbits at a height above the Earths surface equal to half the Earth's radius. Given that the radius of the Earth is 6.4×10^{6} m, and the mass of the Earth is 6.0×10^{24} kg, calculate the gravitational force on the satellite.
6. The weight of a 10 kg mass on the Earth's surface is approximately 98 N, and the radius of the Earth is 6.4×10^{6} m. Use this information to calculate a value for the mass of the Earth.

7 Two lead spheres of radius 50 mm just touch each other. Calculate

(a) the volume of the spheres, in m^3,
(b) the mass of the spheres, if the density of lead is 11000 kg m^{-3},
(c) the gravitational force of attraction between them.

8 If a satellite was placed on the surface of a planet of radius r, it would experience a force of F. Show that if it were put in an orbit at a height of $r/50$ above the planet's surface, the force on the planet would be $0.96F$.