On gravitational fields

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Warm-up problems

1. Express Newton's law of gravitation in symbols, and show how the units of G can be expressed as $\mathrm{m^3\,kg^{-1}\,s^{-2}}$.

2. Define gravitational field strength, and show that the units of gravitational field strength are equivalent to those of acceleration.

3. Define gravitational potential and state its units. What is the link to GPE?

Regular problems

4. Calculate the gravitational pull of the Earth on the Moon, if the mass of the moon is 7.4×10^{22} kg, and the mass of the Earth is 6.0×10^{24} kg, and the distance between their centres is 3.8×10^{8} m.

5. Two lead spheres of radius 50 mm just touch each other. Calculate
   (a) the volume of the spheres, in $\mathrm{m^3}$,
   (b) the mass of the spheres, if the density of lead is 11000 kg $\mathrm{m^{-3}}$,
   (c) the gravitational force of attraction between them.

6. If a satellite was placed on the surface of a planet of radius r, it would experience a force of F. Show that if it were put in an orbit at a height of $r/50$ above the planet's surface, the force on the planet would be $0.96F$.

7. The gravitational field strength on the moon is 1.7 N $\mathrm{kg^{-1}}$. Assuming that the moon is a uniform sphere of radius 1.74×10^{6} m, calculate
   (a) the mass of the moon,
   (b) the gravitational field strength 1.0×10^{6} m above its surface.

8. The moon has a mass of 7.7×10^{22} kg and a radius of 1.7×10^{6} m. Calculate
   (a) the gravitational potential at its surface,
   (b) the gravitational potential at a height of 1.0×10^{6} m above the surface,
   (c) the gravitational potential difference between these two points,
   (d) the work done in moving a 1.5×10^{3} kg space craft from the moon's surface to a height of 1.0×10^{6} m,

(e) the additional work needed in moving the spacecraft from this height until it totally escapes the moon's gravitational field.

9. (a) What is the minimum gravitational potential energy which an object of mass m needs to gain if it is to completely escape the gravitational field of a planet of mass M and radius R from its surface?

   (b) Presuming the object has this energy only from the kinetic energy at the start of its motion, show that the minimum initial speed for escape is

$$v_e = \sqrt{\frac{2GM}{R}}.$$

   (c) Show that this can be rewritten $v_e = \sqrt{2g_s R}$, where g_s is the surface gravitational field strength.

Extension problems

10. Why could Newton's law of gravitation *not* be

$$F = G\frac{{m_1}^2 m_2}{r^2}?$$

11. A man is able to jump vertically 1.5 m on Earth. What height should he be able to jump on a planet of one third of the density of the Earth and one half of its radius?