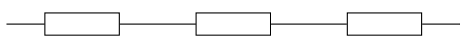
On resistor combinations

A.C. Norman
ACN.Norman@radley.org.uk

Warm-up problems

1. What are the rules for combining the resistances of resistors connected in series and parallel?

   For resistors in series, to find the equivalent resistance R_T of the series combination, we simply add the resistances up:

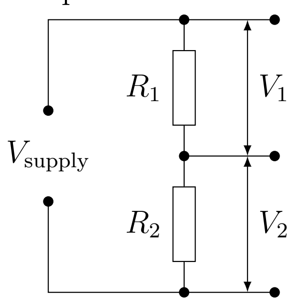
   

   $$R_T = R_1 + R_2 + \dots .$$

   For resistors in parallel, the combined resistance R_T (which is less than the resistance of the smallest resistor) is given by

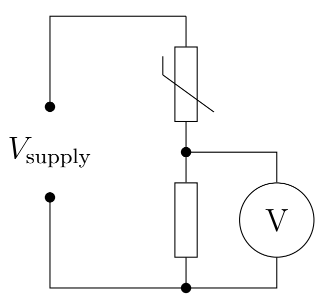
   $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots +$$

2. Draw a circuit diagram for the common configuration of resistors known as a *potential divider*, and explain how it works.

   A potential or voltage divider provides a convenient way of getting a variable voltage from a fixed voltage supply. The voltages appearing across the resistors R_1 and R_2 are in the ratio of their resistances, i.e. $V_1/V_2 = R_1/R_2$.

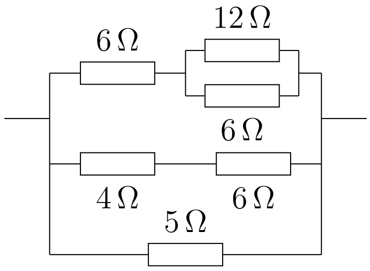
3. Find out how a potential divider is used in a sensing circuit to measure temperature, and draw a possible circuit diagram of such an arrangement.

   The reading on the voltmeter can be calibrated to give an indication of the temperature at the thermistor. Most thermistors are negative temperature coefficient (NTC), in which the resistance decreases with increasing temperature; therefore the voltage across the bottom resistor will increase with increasing T as it gets a greater share of the supply voltage.

Regular problems

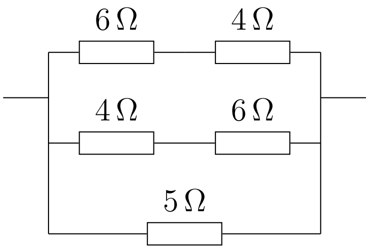
4. Find the combined resistance of the following resistors, showing your step-by-step working carefully.

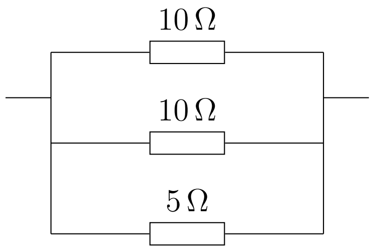


First calculate the resistance of the parallel $12\,\Omega$ and $6\,\Omega$ resistors in the top branch:

$$\frac{12\,\Omega \times 6\,\Omega}{12\,\Omega + 6\,\Omega} = 4\,\Omega.$$

The three branches are now all series combinations, which can be easily worked out by addition:





The final combined resistance R_f can now be calculated from the resistances of the three parallel branches:

$$\frac{1}{R_f} = \frac{1}{10\,\Omega} + \frac{1}{10\,\Omega} + \frac{1}{10\,\Omega} \quad (1)$$
$$= 0.4\,\Omega^{-1} \quad (2)$$
$$R_f = \frac{1}{0.4\,\Omega^{-1}} \quad (3)$$
$$= 2.5\,\Omega. \quad (4)$$

5. Use the rule for combinations of parallel resistors to

(a) prove that, for two resistors with resistances R_1 and R_2 connected in parallel, their combined resistance is $\dfrac{R_1 R_2}{R_1 + R_2}$;
Starting with the rule for resistors in parallel,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots +,$$

in this case, there are only two resistors, so we can write

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Now rearranging:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \quad (5)$$
$$= \frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2} \quad (6)$$
$$= \frac{R_1 + R_2}{R_1 R_2} \quad (7)$$
$$R_T = \frac{R_1 R_2}{R_1 + R_2).} \quad (8)$$

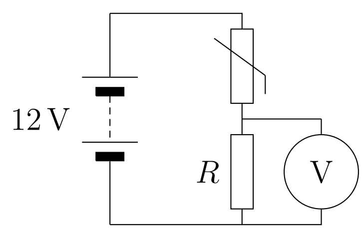
(b) prove that, for N resistors of equal resistance R in parallel, their combined resistance is R/N.
Again, we start with the rule for resistors in parallel,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots +,$$

in this case, we have N resistors, all with resistance R:

$$\frac{1}{R_T} = \underbrace{\frac{1}{R} + \frac{1}{R} + \ldots + \frac{1}{R}}_{N} \quad (9)$$
$$= N\left(\frac{1}{R}\right) \quad (10)$$
$$= \frac{N}{R} \quad (11)$$
$$R_T = \frac{R}{N} \quad (12)$$

6. (AQA) A thermistor is connected in series with a resistor, R, and battery of emf 6.0 V and negligible internal resistance.



When the temperature is 50 °C the resistance of the thermistor is 1.2 kΩ. The voltmeter connected across R reads 1.6 V.

(a) Calculate the p.d. across the thermistor.

$$6.0\,\text{V} - 1.6\,\text{V} = 4.4\,\text{V}.$$

(b) Calculate the current in the circuit.
For the thermistor

$$I = \frac{V}{R} \tag{13}$$
$$= \frac{4.4\,\text{V}}{1.2 \times 10^3\,\Omega} \tag{14}$$
$$= 3.7 \times 10^{-3}\,\text{A}. \tag{15}$$

(c) Calculate the resistance of R quoting your answer to an appropriate number of significant figures.
The current in R is the same as in the thermistor, as the two are in series.

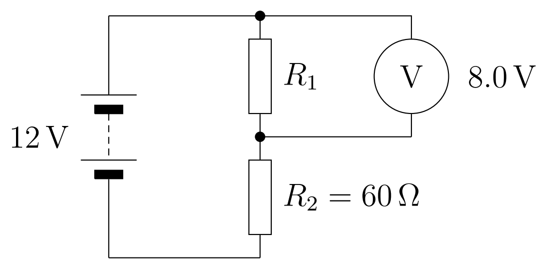
$$R = \frac{V}{I} \tag{16}$$
$$= \frac{1.6\,\text{V}}{3.7 \times 10^{-3}\,\text{A}} \tag{17}$$
$$= 440\,\Omega. \tag{18}$$

Alternatively, the ratio of the voltages across R and the thermistor are in the ratio of their resistances

$$\frac{V_R}{V_\text{therm}} = \frac{R}{R_\text{therm}} \tag{19}$$
$$R_\text{therm} = R \times \frac{V_\text{therm}}{V_R} \tag{20}$$
$$= 1.2 \times 10^3\,\Omega \times \frac{1.6\,\text{V}}{4.4\,\text{V}} \tag{21}$$
$$= 440\,\Omega. \tag{22}$$

7. (AQA question, adapted) Two resistors, R_1 and R_2 are connected in series with a battery of e.m.f. 12 V and negligible internal resistance. If a voltmeter is connected across R_1, the reading on the voltmeter is 8.0 V, and the resistance of R_2 is 60 Ω.

(a) Calculate the current in the circuit.



For R_2

$$I = \frac{V}{R} \tag{23}$$
$$= \frac{4.0\,\text{V}}{60\,\Omega} \tag{24}$$
$$= 67 \times 10^{-3}\,\text{A}. \tag{25}$$

(b) Calculate the resistance of R_1.
The ratio of the voltages across R_1 and R_2 are in the ratio of their resistances

$$\frac{R_1}{R_2} = \frac{V_1}{V_2} \tag{26}$$

$$R_1 = R_2 \times \frac{V_1}{V_2} \qquad = 60\,\Omega \times \frac{8.0\,\text{V}}{4.0\,\text{V}} \tag{27}$$

$$= 120\,\Omega. \tag{28}$$

(c) Calculate the charge passing through the battery in 2.0 minutes. Give an appropriate unit for your answer.

$$Q = It \tag{29}$$

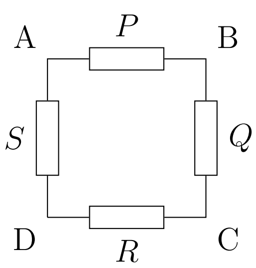
$$= 67 \times 10^{-3}\,\text{A} \times 2.0\,\text{minute} \times \frac{60\,\text{s}}{1\,\text{minute}} \tag{30}$$

$$= 8\,\text{C}. \tag{31}$$

(d) R_2 is now replaced with a thermistor. State and explain what will happen to the reading on the voltmeter as the temperature of the thermistor increases.
The reading on the voltmeter will increase as the temperature increases. With increasing temperature, the resistance of the thermistor decreases (most thermistors are negative temperature coefficient, NTC). This means it will get a smaller share of the voltage—the voltage across the thermistor will decrease—meaning that the reading on the voltmeter (measuring the voltage across R_1) will increase.

Extension problems

8. (based on a CEA Advanced extension award question) Four resistors of value 1.0 Ω, 2.0 Ω, 3.0 Ω and 4.0 Ω are used to make this circuit.



The resistance is measured between pairs of terminals in turn, with the following results:

between terminals A and B	1.6 Ω
between terminals B and C	0.9 Ω
between terminals C and D	2.4 Ω
between terminals D and A	2.1 Ω

Deduce the resistance of each of the resistors P, Q, R and S. Show your reasoning clearly.
In this problem, R_{AB}, R_{BC}, R_{CD}, and R_{DA} are known, and R_P, R_Q, R_R, and R_S are unknown.

Using the rules for combining resistors, we can write

$$R_{AB} = \frac{R_P \times (R_Q + R_R + R_S)}{R_P + R_Q + R_R + R_S} \tag{32}$$

$$R_{BC} = \frac{R_Q \times (R_R + R_S + R_P)}{R_P + R_Q + R_R + R_S} \tag{33}$$

$$R_{CD} = \frac{R_R \times (R_S + R_P + R_Q)}{R_P + R_Q + R_R + R_S} \tag{34}$$

$$R_{DA} = \frac{R_S \times (R_P + R_Q + R_R)}{R_P + R_Q + R_R + R_S} \tag{35}$$

We know that the values of $\{R_P, R_Q, R_R, R_S\}$ are $\{1.0\,\Omega, 2.0\,\Omega, 3.0\,\Omega, 4.0\,\Omega\}$. Computing any of the sums above will allow us to identify one of the resistors. e.g.

$$\frac{1.0\,\Omega \times (2.0\,\Omega + 3.0\,\Omega + 4.0\,\Omega)}{1.0\,\Omega + 2.0\,\Omega + 3.0\,\Omega + 4.0\,\Omega} \tag{36}$$

$$= \frac{1.0\,\Omega \times (9.0\,\Omega)}{10.0\,\Omega} \tag{37}$$

$$= 0.9\,\Omega = R_{BC}. \tag{38}$$

allows us to identify that $R_Q = 1.0\,\Omega$ (by comparing the sum with the expression 33).

We can continue this by changing the sum, as follows:

$$\frac{2.0\,\Omega \times (1.0\,\Omega + 3.0\,\Omega + 4.0\,\Omega)}{1.0\,\Omega + 2.0\,\Omega + 3.0\,\Omega + 4.0\,\Omega} \tag{39}$$

$$= \frac{2.0\,\Omega \times (8.0\,\Omega)}{10.0\,\Omega} \tag{40}$$

$$= 1.6\,\Omega = R_{AB}, \tag{41}$$

implying by comparison with 32 that $R_P = 2.0\,\Omega$.

$$\frac{3.0\,\Omega \times (1.0\,\Omega + 2.0\,\Omega + 4.0\,\Omega)}{1.0\,\Omega + 2.0\,\Omega + 3.0\,\Omega + 4.0\,\Omega} \tag{42}$$

$$= \frac{3.0\,\Omega \times (7.0\,\Omega)}{10.0\,\Omega} \tag{43}$$

$$= 2.1\,\Omega = R_{DA}, \tag{44}$$

implying by comparison with 35 that $R_S = 3.0\,\Omega$, and (by elimination of the other 3 possibilities), $R_R = 4.0\,\Omega$. To check this, use 34 as follows:

$$R_{CD} = \frac{R_R \times (R_S + R_P + R_Q)}{R_P + R_Q + R_R + R_S} \tag{45}$$

$$= \frac{4.0\,\Omega \times (1.0\,\Omega + 2.0\,\Omega + 3.0\,\Omega)}{1.0\,\Omega + 2.0\,\Omega + 3.0\,\Omega + 4.0\,\Omega} \tag{46}$$

$$= \frac{4.0\,\Omega \times (6.0\,\Omega)}{10.0\,\Omega} \tag{47}$$

$$= 2.4\,\Omega \tag{48}$$

as required.