On electrical power

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Problem 10 is not original. Problem 9 owes a debt to G. Polya.

Warm-up problems

1. Write down three formulas for the power in an electrical component. Remember to include what the usual units are and what each symbol means. When is each useful?

   $P = IV$, $P = I^2R$, $P = \frac{V}{R}$ are three formulas for the power in an electrical component. P represents the power dissipated in the component, and has the unit of watt (W). I represents the current through the component, and has the unit of ampere (A). V represents the voltage across the component, and has the unit of volt (V). R represents the electrical resistance of the component, and has the unit of ohm (Ω).

2. Link the following terms with their units.

   (a) current
   (b) e.m.f.
   (c) p.d.
   (d) resistance
   (e) charge
   (f) energy
   (g) power

   i. joule
   ii. coulomb
   iii. watt
   iv. volt
   v. ohm
   vi. ampere
   vii. volt

   (a)–vi; (b)–iv; (c)–vii; (d)–v; (e)–ii; (f)–i; (g)–iii.

3. Why does a lamp change more electrical energy per second into heat and light than the wires connecting it to the supply? Use the term *resistance* in your answer.

   The current through the lamp and the wires connecting it to the supply is the same, but the power dissipated in each depends on the resistance ($P = I^2R$), and so the bulb dissipates more electrical energy as heat and light, because it has a much greater resistance than the wires.

Regular problems

4. (a) What is the power of a lamp rated at 12 V 2 A?

$$\begin{aligned} P &= IV && (1)\\ &= 2\,\text{A} \times 12\,\text{V} && (2)\\ &= 24\,\text{W} && (3) \end{aligned}$$

(b) How much energy is radiated per second from a 6 V 0.5 A lightbulb?

$$\begin{aligned} P &= IV && (4)\\ &= 0.5\,\text{A} \times 6\,\text{V} && (5)\\ &= 3\,\text{W} && (6) \end{aligned}$$

(c) A lightbulb is labelled 12 V 36 W. When used on a 12 V supply,

i. what current will it take?

$$\begin{aligned} I &= \frac{P}{V} && (7)\\ &= \frac{36\,\text{W}}{12\,\text{V}} && (8)\\ &= 3\,\text{A} && (9) \end{aligned}$$

ii. what is its resistance?

$$\begin{aligned} R &= \frac{V}{I} && (10)\\ &= \frac{12\,\text{V}}{3\,\text{A}} && (11)\\ &= 4\,\Omega && (12) \end{aligned}$$

5. (a) Why are thick wires rather than thin wires used to carry high currents?
The power dissipated in the wire depends on its resistance $P \propto R$. As the resistance depends on the thickness of the wire $R \propto \frac{1}{A} \propto \frac{1}{t^2}$, it is important that wires used to carry high currents are thick enough: if they are not, too much power is dissipated (as heat) in the wire, and there is a danger of overheating, melting the insulation, causing short circuits and / or fires.

(b) Will a lamp with a thick filament draw more current or less current (if screwed into a mains socket) than a lamp with a thin filament?
In this case, both lamps are screwed into a mains socket, so they will have the same voltage of 230 V across them. This is a good example of where $P = \frac{V^2}{R}$ is the best power formula to use, and the lower resistance of the lamp with a thicker filament will mean that it draws more current and dissipates more power.

6. (a) What is the electrical power dissipated in a 100 Ω resistor carrying a current of 50 mA?

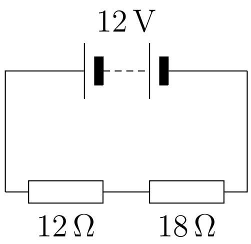
$$P = I^2 R \tag{13}$$
$$= \left(50 \times 10^{-3}\,\mathrm{A}\right)^2 \times 100\,\Omega \tag{14}$$
$$= 0.25\,\mathrm{W}. \tag{15}$$

(b) Would a 0.5 W rating be suitable for a 10 kΩ resistor through which a current of 10 mA flows? If it is not, what rating would be suitable?

$$P = I^2 R \tag{16}$$
$$= \left(10 \times 10^{-3}\,\mathrm{A}\right)^2 \times 10 \times 10^3\,\Omega \tag{17}$$
$$= 1\,\mathrm{W}. \tag{18}$$

Thus a 0.5 W rating would not be suitable (the expected power is too high). A rating of more than 1 W should be used.

7. A 12 Ω and 18 Ω resistor are connected in series with a 12 V battery.



(a) What is the current drawn from the battery?

$$I = \frac{V}{R} \tag{19}$$
$$= \frac{12\,\mathrm{V}}{12\,\Omega + 18\,\Omega} \tag{20}$$
$$= 0.4\,\mathrm{A}. \tag{21}$$

(b) What is the power dissipated in the 12 Ω resistor?

$$P = I^2 R \tag{22}$$
$$= (0.4\,\mathrm{A})^2 \times 12\,\Omega \tag{23}$$
$$= 1.9\,\mathrm{W}. \tag{24}$$

(c) Which of the two resistors dissipates the greater power?
As the current is the same in both resistors, and $P = I^2 R$, the resistor with the greater resistance has the larger power dissipation, i.e. the 18 Ω resistor in this case.

(d) The two resistors are now connected to the battery in parallel. What is the new (i) current and (ii) drawn from the battery, and (iii) which of the two resistors now dissipates the greatest power?

The voltage is now the same across the two resistors (12 V). The current drawn from the battery is $I = \frac{V}{R_T} = \frac{12\,\text{V}}{12\,\Omega} + \frac{12\,\text{V}}{18\,\Omega} = 1.7\,\text{A}$. As the voltage is the same in both resistors, and $P = \frac{V^2}{R}$, the resistor with the smaller resistance has the larger power dissipation, i.e. the $12\,\Omega$ resistor in this case.

8. Should a kettle element have a lower or higher resistance in order to boil the water more quickly? Justify your answer.

A kettle element should have a lower resistance in order to boil water more quickly. It is connected to the mains supply (230 V a.c.) and since V is unalterable and $P = \frac{V^2}{R}$, a smaller resistance should be used. One way to achieve this is to have a number of coils of resistive wire (which will get hot when current flows) in parallel.

Extension problems

9. Question 7 was what is known as a problem "in numbers". You should have found that the resistor which used greater power in series was the less powerful one in parallel. Look back at your solution to that problem and see whether

(a) Can you now see this result *at a glance*? Why does it arise (what is the main criterion in series and parallel?)

The main criterion is that the current is the same for both resistors in series (so $P = I^2R$ is more useful), and the voltage is the same in parallel (so $P = \frac{V^2}{R}$ is more useful).

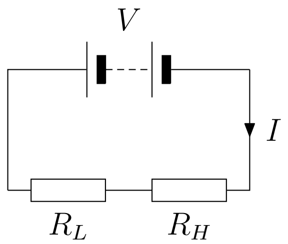
in series	current through each resistor is the same	$P = I^2R$	...shows that P dissipated is greater for the resistor with higher resistance
in parallel	voltage across each resistor is the same	$P = \frac{V^2}{R}$	...shows that P dissipated is greater for the resistor with lower resistance

(b) Can you turn the problem into a problem "in letters"? [*Use unambiguous and easy to remember notation, e.g. R_L and R_H rather than R_1 and R_2.*]

First let's consider the advantages of turning a problem into a problem "in letters". You should know from your own experience that problems "in letters" have a great advantage over purely numerical problems; if the problem is given "in letters" it is accessible to several tests (and more interesting tests) to which a problem "in numbers" is not susceptible at all (for example a test by dimension or *dimensional analysis*), and passing these tests allow us to gain confidence in our answer.

These advantages are so great that is is often worth introducing suitable notation into a problem to make it into a problem "in letters". In doing this—if for the numbers in a problem we substitute letters—we *generalize*. We obtain a more general problem than the first. Such generalization may be very useful. Passing from a problem "in numbers" to another one "in letters" we gain access to new procedures; we can vary the data, and, in doing so, we may check our results in various ways. The more general problem may thus be easier to solve. This sounds paradoxical, but sometimes the main achievement in solving the special problem is to invent the general problem.

After doing this, the tests mentioned before can be made not only to the final result but also to intermediate results. The solution that we make is likely to be more transferable. The terms of a symbolic equation acquire significance, and are linked up with various facts. The formula stands a better chance of being remembered, and thus the knowledge gained in the solution is consolidated. Finally, the solution can more easily be transferred to other problems.

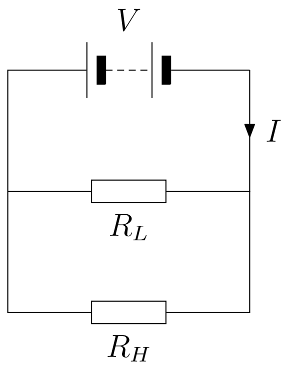


$$I = \frac{V}{R_L + R_H}. \tag{25}$$

$P = I^2 R$, so

$$P_L = \frac{V R_L}{R_L + R_H} \text{ and } P_H = \frac{V R_H}{R_L + R_H}. \tag{26}$$

Notice that $P_H > P_L$ since $R_H > R_L$.



$$I = \frac{V}{R_L} + \frac{V}{R_H}. \tag{27}$$

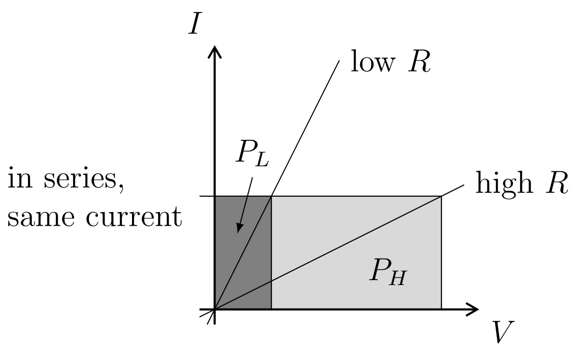
$P = \frac{V^2}{R}$, so

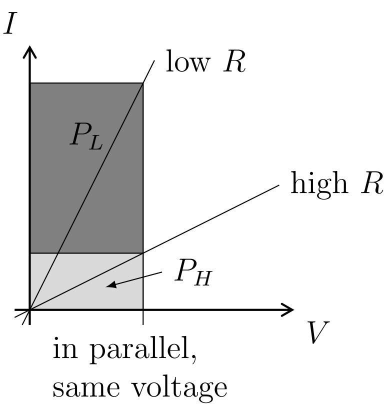
$$P_L = \frac{V^2}{R_L} \text{ and } P_H = \frac{V^2}{R_H}. \tag{28}$$

Notice that $P_L > P_H$ since $\frac{1}{R_L} > \frac{1}{R_H}$.
The total power drawn from the battery is given (by $P = IV$ or by summation of resistor powers) as $P = \frac{V^2}{R_L} + \frac{V^2}{R_H}$.

(c) Can you make a *visual proof* of the result?
For an ohmic conductor such as a resistor, the current–voltage characteristic is a straight line passing through the origin (i.e. current is directly proportional to voltage).

At a particular point on the $I - V$ characteristic, the power dissipated is given by IV, the product of the current and the voltage values. This can be represented by a rectangle with its corners at the origin and the point, and bounded by the graph's axes:





10. Downstairs in a house are three identical on-off switches. One of them controls the lamp in the attic. The puzzle is to work out which switch controls the lamp.

 The rules are as follows. You are allowed to manipulate the switches all you like, and then you are allowed only one trip to the attic. How do you do it?

 I first saw this problem as a *Monday puzzle* set by Alex Bellos in the Guardian, but it's a classic puzzle. He said that, despite having challenged many people with this puzzle over the years, only one person has solved it straightaway, right there in front of him: Jo Nesbø, the Norwegian crime writer, who took a "good few minutes" to solve it. Even knowing that the authors of detective fiction are going to be pretty handy at logical deduction, Bellos was immensely impressed. Here follows Bellos' solution.

 First let's simplify the question.

 If there were only two switches, A and B, the solution is trivial. We switch A on and B off. We head to the attic. If the lamp is on, A is the correct switch, and if the light is off, it's B.

 It gets tough with three switches, A, B and C. What are the possible combinations?

 (a) All switches off, or all switches on.
 (b) One switch on, two off.
 (c) Two switches on, one off.

 None of these combinations provide us with a solution, since there is no way to distinguish between two switches in the same position.

 It looks like we are stuck. Who is going to save the day? According to Bellos:

> The date is lost in the midst of time. The place is a literary festival.
>
> An unshaven Scandinavian man is sitting on a sofa in a Glasgow hotel in front of some courtesy nibbles.
>
> It's Jo Nesbø, the nabob of Nordic noir.
>
> On hearing I am a maths author he asks me for a puzzle. I give him the Three Switches.
>
> "There is not enough information just from the switches," he tells me matter-of-factly. "You need to get information from some other way."
>
> After a few minutes of intense thought ... the lightbulb in his head switches on.
>
> Heat, he says. When you switch on a lamp the bulb gets warm.
>
> CORRECT!

 (In the Guardian website comments section, many people complained that the question should have specified that the bulb was not LED, since LED bulbs don't get hot. Bellos explains that this would have given away the answer in the question, but also because

LED bulbs do produce some heat. A lot less heat than with incandescent bulbs but still enough to feel a difference.)

The solution is as follows:

- Switch on A and B for a minute, and then switch off B.
- Go to the attic.
- If the lamp is on, A is the answer. If the lamp is off but warm, it's B, and if it is off and cold, it's C.

Upon solving it, Nesbø said that he was able to second-guess the puzzle because it does exactly what he does as a crime writer. One of the skills of writing a suspenseful thriller is to lead the reader in completely the wrong direction. And the three switches puzzle points the solver in completely in the wrong direction. You begin by considering clever manipulations of the switches, as if the answer is based on logic alone.

Yet the solution involves an understanding of the physical world.

It is very hard when solving a puzzle—as it is in life—to change one's mindset once it is already fixed. The lesson here is that often it is almost impossibly hard to think of the simplest, most obvious thing.

At least one comment suggested another, less well known solution:

- Switch A on and wait for as long as you need to be sure that the bulb burns out. This could be a couple of years if its a low energy, long life lamp.
- Switch on B and go upstairs.
- If the light is burnt out A is the answer, if it is on it's B and if it's off it is C.

But this would make for a very slow plot!