On current electricity

A.C. Norman
ACN.Norman@radley.org.uk

Warm-up problems

1. What are the units of charge, current, voltage and resistance? Express each of these units in SI base units.

Symbol	Quantity	Unit	SI base units
Q	charge	C	$\mathrm{A\,s}$
I	current	A	A
V	voltage	V	$\mathrm{kg\,m^2\,s^{-3}\,A^{-1}}$
R	resistance	R	$\mathrm{kg\,m^2\,s^{-3}\,A^{-2}}$

2. What are the other two alternative names for *voltage*? Give the equation linking voltage, energy and charge.

   In some books, *voltage* is also called *potential difference*, or *electromotive force* (probably for jolly good reasons: potential difference makes more sense if you are familiar with potential from fields, and electromotive force is a good term for use with batteries). They are all measured with the unit volt, though. The equation which links voltage, energy and charge is

$$\text{voltage} = \frac{\text{energy}}{\text{charge}}$$

3. Give the equations linking

   (a) current to charge and time,

$$\text{current} = \frac{\text{amount of charge passed}}{\text{time taken}}$$

   (b) resistance to current and voltage.

$$\text{current} = \frac{\text{voltage}}{\text{resistance}}$$

Regular problems

4. Why are *electrons*, rather than *protons*, the principal charge carriers in a metal wire?

   Protons cannot move from their fixed position because they are trapped by electrostatic forces at the centre of an atom. Electrons, on the other hand, are delocalized in metal wires into a sea of electrons which are free to move fairly freely (like atoms in a gas) and whizz around the space (there is lots of empty space in solid matter). If a voltage is applied, it is they that move (drift) in response.

5. (a) What is the current in a circuit if the charge passing each point is (i) 10 C in 2 s, (ii) 20 C in 40 s, (iii) 240 C in 2 minute?

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \tag{1}$$
$$= \frac{10\,\mathrm{C}}{2\,\mathrm{s}} \tag{2}$$
$$= 5\,\mathrm{A}. \tag{3}$$

   The same formula gives (ii) 0.5 A, (iii) 2 A.

   (b) If the current through a lamp is 5 A, what charge passes in (i) 1 s, (ii) 10 s, (iii) 1 minute?

$$Q = \int I\,\mathrm{d}t, \text{ so for constant current,} \tag{4}$$
$$Q = It \tag{5}$$
$$= 5\,\mathrm{A} \times 1\,\mathrm{s} \tag{6}$$
$$= 5\,\mathrm{C}. \tag{7}$$

   The same formula gives (ii) 50 C, (iii) 300 C.

6. The voltage across a lamp is 12 V. How much electrical energy is changed into heat and light when

   (a) a charge of 1 C passes through it,

$$V = \frac{E}{Q} \tag{8}$$
$$E = QV \tag{9}$$
$$= 1\,\mathrm{C} \times 12\,\mathrm{V} \tag{10}$$
$$= 12\,\mathrm{J}. \tag{11}$$

   (b) a charge of 5 C passes through it,

$$E = QV \tag{12}$$
$$= 5\,\mathrm{C} \times 12\,\mathrm{V} \tag{13}$$
$$= 60\,\mathrm{J}. \tag{14}$$

(c) a current of 2 A passes through it for 10 s?

$$E = QV \tag{15}$$
$$= ItV \tag{16}$$
$$= 2\,\text{A} \times 10\,\text{s} \times 12\,\text{V} \tag{17}$$
$$= 240\,\text{J}. \tag{18}$$

7. (a) What is the voltage across a 220 Ω resistor when a current of 3 mA flows through it?

$$V = IR \tag{19}$$
$$= 3\,\text{mA} \times 220\,\Omega \tag{20}$$
$$= 0.66\,\text{V}. \tag{21}$$

(b) The voltage across a 2.7 kΩ resistor is 5.4 V. What current flows?

$$I = \frac{V}{R} \tag{22}$$
$$= \frac{5.4\,\text{V}}{2.7\,\text{k}\Omega} \tag{23}$$
$$= 2.0\,\text{mA}. \tag{24}$$

(c) Calculate the resistance of a resistor if a voltage of 9 V causes a current of 1.5 mA to flow through is.

$$R = \frac{V}{I} \tag{25}$$
$$= \frac{9\,\text{V}}{1.5\,\text{mA}} \tag{26}$$
$$= 6.0\,\text{k}\Omega. \tag{27}$$

(d) A current of 0.4 mA flows through a 10 kΩ resistor. What is the voltage across its ends?

$$V = IR \tag{28}$$
$$= 0.4\,\text{mA} \times 10\,\text{k}\Omega \tag{29}$$
$$= 4\,\text{V}. \tag{30}$$

8. (a) If the voltage across a circuit is held constant while the resistance doubles, what change occurs in the current?
$I = \frac{V}{R}$, so if resistance increases to $2R$, the current will reduce to $I/2$.

(b) If the resistance across a circuit halves while the voltage increases by a factor of three, what change occurs in the current?
$I = \frac{V}{R}$, so if resistance increases to $R/2$, whilst the voltage goes up to $3V$, the current will increase to $6I$.

9. (a) What is the combined resistance of the following resistors connected in parallel: $220\,\Omega$, $100\,\Omega$, $470\,\Omega$?

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{31}$$
$$= \frac{1}{220\,\Omega} + \frac{1}{100\,\Omega} + \frac{1}{470\,\Omega} \tag{32}$$
$$= 1.667 \times 10^{-2}\,\Omega^{-1} \tag{33}$$
$$R = \frac{1}{1.667 \times 10^{-2}\,\Omega^{-1}} \tag{34}$$
$$= 60\,\Omega. \tag{35}$$

(b) Calculate the the combined value of the following resistors connected in series: $33\,\text{k}\Omega$, $18\,\text{k}\Omega$, $4.7\,\text{k}\Omega$.

$$R = R_1 + R_2 + R_3 \qquad = 33\,\text{k}\Omega + 18\,\text{k}\Omega + 4.7\,\text{k}\Omega = 56\,\text{k}\Omega. \tag{36}$$

If a battery supplies a current of $215\,\mu\text{A}$ through the resistors when they are connected to it, what is the battery voltage?

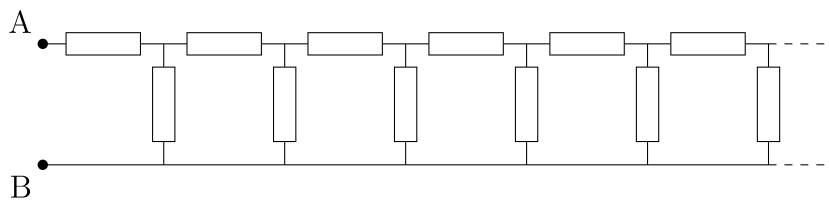
$$V = IR \tag{37}$$
$$= 215\,\mu\text{A} \times 56\,\text{k}\Omega \tag{38}$$
$$= 12\,\text{V}. \tag{39}$$

Extension problems

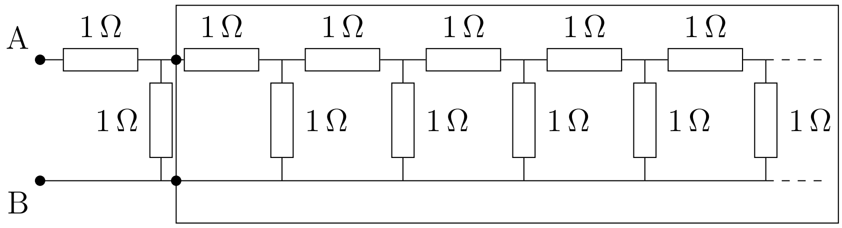
10. A $10\,\text{k}\Omega$ and a $6\,\text{k}\Omega$ resistor are connected in series. If both resistors have a manufacturing tolerance of $\pm 10\,\%$, what, approximately, are the maximum and minimum values of resistance we should expect to measure across the combination?

    This question is about combining uncertainties. The $10\,\text{k}\Omega$ could have a true resistance of $10(1)\,\text{k}\Omega$. The other resistor might have a resistance of $6.0(6)\,\text{k}\Omega$. When combining these in series, we add the resistances, meaning we should add the absolute uncertainties, giving $16(2)\,\text{k}\Omega$. The maximum value we should expect to read (we presume the uncertainty associated with the resistance measurement is *much* smaller than 10%) is therefore $18\,\text{k}\Omega$ and the minimum value is therefore $14\,\text{k}\Omega$.

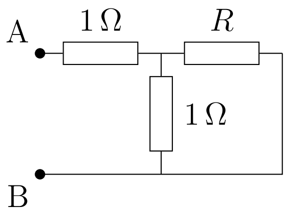
11. In the following infinite network of $1\,\Omega$ resistors, what is the resistance (as measured by an ohmmeter) between A and B?



This resistive network contains a copy of itself (enclosed in the box):



Call R the resistance of the network inside the box, measured between the two dots as the terminals. Then the original network, which also has resistance R, is



It is a $1\,\Omega$ resistance in series with the parallel combination of $1\,\Omega$ and R. So

$$R = 1 + \frac{R}{1+R},$$

or $R^2 - R - 1 = 0$. The positive solution is

$$R = \frac{1+\sqrt{5}}{2} \approx 1.618,$$

which is the Golden Ratio.

An alternative, direct method is the following continued fraction that accounts for the infinite cascase of series and parallel resistors:

$$R = 1 + \frac{1}{1 + \frac{1}{1+\ldots}}.$$

This famous continued fraction converges (slowly) to the Golden Ratio. (One special feature of the Golden Ratio is that it has the the slowest-converging continued fraction of any real number.)