Resistivity

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An important property of solid matter is its *mass*, which tells us its resistance to acceleration when a force is applied. However, the mass of a substance is not the quantity to estimate first, because it is not invariant under simple changes. For example, a thick slab of material has more mass than a thin one. Similarly, a shorter length of material accelerates more easily than a longer length. The property independent of the shape or size of the substance—invariant to these changes—is the *density*.

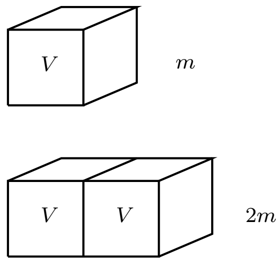

Figure 1: Imagine an object of mass m having a simple cubic shape of volume V. If another such object is placed alongside, the two together will have volume $2V$ and mass $2m$.

As shown in figure 1, the mass of an object is proportional to its volume. This allows us to write

$$m \propto V, \text{ so}$$
$$m = \rho V,$$

where ρ is a factor with dimensions. This factor gives us a quantity $\rho = \frac{m}{V}$ which will be independent of the volume, and only depends on the material. Knowing the density tells us more about the material (as opposed to the particular object), and it is thus a more broadly useful material property.

Electrical *resistance* is an important electrical property of matter, which tells us its resistance to current when a voltage is applied. A similar argument to the one above used to define density can be used to find a quantity analogous to electrical resistance for a material.

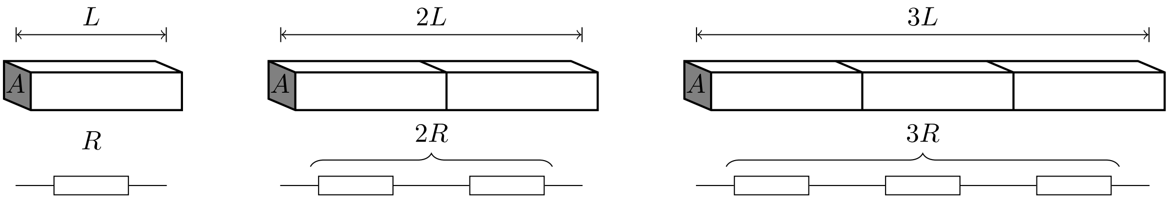

Figure 2: Imagine a block of material with resistance R, having a simple square cross section of area A and length L. If more such blocks are added end-to-end, this has the same effect as connecting resistors in series with each other, so the total resistance of such a combination will be the sum of the resistances of each block as shown.

To find out how the resistance of a block of material depends on its length, imagine putting a number of blocks of length L end-to-end, as shown in figure 2. In this way it is easily seen that the resistance of a block of material is proportional to its length,

$$R \propto l.$$

The resistance of a block of material also depends on its cross sectional area A. To find out how, imagine putting a number of blocks of cross sectional area A side-by-side, as shown in figure 3. This has the effect of connecting the resistor-like blocks in parallel with each other, thus showing that the resistance of a block of material is inversely proportional to its cross sectional area,

$$R \propto \frac{1}{A}.$$

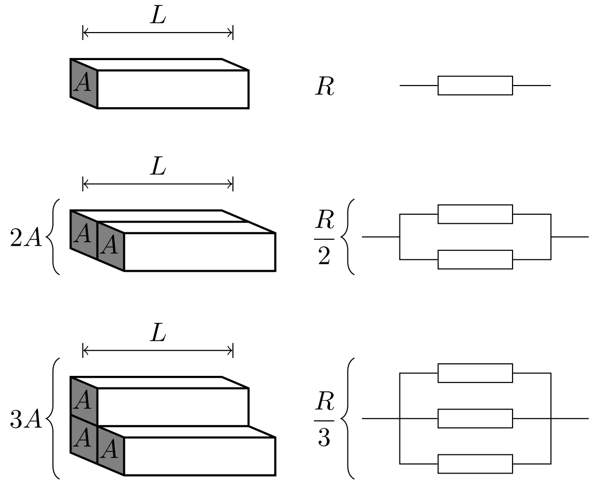

Figure 3: If blocks of material having a simple square cross section of area A and length L (each with resistance R) are placed side-by-side, this has the same effect as connecting resistors in parallel with each other. This means that the cross sectional area of such a combination of blocks will be the sum of the cross sectional areas of each block, and the equivalent resistance will be the resistance of one block divided by the number of blocks, as shown.

Combining the dependences of the resistance on the length l and the cross sectional area A, we can write

$$R \propto \frac{l}{A}, \text{ so}$$
$$R = \frac{\rho l}{A},$$

where ρ is a factor with dimensions. This factor gives us a quantity $\rho = \frac{RA}{l}$ which is independent of the shape of the block. Let's call it *resistivity*, as it is the version of resistance most useful for comparing materials (by analogy with density, the 'materials' version of mass). Just as the dimensions of density can be obtained using its definition by dividing those of mass by those of volume (so in SI units its unit is $\mathrm{kg\,m^{-3}}$), to find out the dimensions of resistivity its definition $\rho = \frac{RA}{l}$ can be used, e.g. in SI units the unit of resistivity is the ohm metre ($\Omega\,\mathrm{m}$).

As we have seen, the resistance of a wire is not a property of the wire alone; it depends on the wire's length and area as well. Resistivity fixes this problem; it depends on only the substance.

Material	$\rho\,/\,\Omega\,\mathrm{m}$
Silver	1.59×10^{-8}
Copper	1.68×10^{-8}
Aluminium	2.82×10^{-8}
Iron	9.72×10^{-8}
Carbon	3.0×10^{-5}
Silicon	2.3×10^{3}
PVC	10^{14}

Table 1: The resistivities of different materials at room temperature. The lower the resistivity of a particular material, the better it is at conducting electrical current.