# *Current and charge carriers*

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Electrical currents which we can measure on a large everyday scale (macroscopic) are a direct consequence of the movement of individual charge carriers on an atomic scale (microscopic).

## *Charge carrier number density*

*Estimate the number density of free (conduction) electrons in a copper wire* Among books teaching the art of approximation, a classic is *Consider a Spherical Cow*, so named because a sphere is a much simpler shape than a cow. An even simpler shape is a cube. Thus, a powerful form of lumping is to replace complex shapes by a comparably sized cube. With this idea, we can estimate the number density of free electrons in a metal such as copper.

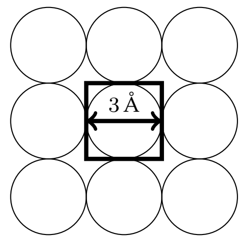


Figure 1: A crude 'cartoon' picture of copper at the atomic scale, with an approximating cube centred on a copper atom.

Each atom is a complex, ill-defined shape, but pretend that it is a cube. Because the atoms touch, the number density of free electrons in the substance is approximately the number density of free electrons in one (approximating) cube:

$$n = \frac{\text{number of electrons, } N}{\text{Volume, } V} \approx \frac{\text{number of free electrons per atom}}{\text{volume of the lumped cube}}.$$

Let's guess that each atom of copper has about one free electron. To find the cube's volume, make each cube's side be a typical atomic diameter[1] of $a \approx 3\,\text{Å}$. This size is based on the fact that the diameter of the smallest atom, hydrogen, is roughly $1\,\text{Å}$. The number density of free electrons in copper then becomes

$$n \approx \frac{1}{(3\,\text{Å})^3} = 3 \times 10^{28}\,\text{m}^{-3}.$$

[1] The ångström (Å) = $10^{-10}$ m is a good unit for atomic sizes, as a typical atom has a size of a few Å. Such modest numbers are easy to remember and handle.

The accepted value is about $8.5 \times 10^{28}\,\text{m}^{-3}$, so amazingly we are within a factor of 10 despite our bold appoximations.

*Knowing the free electron number density, how fast do electrons move along a wire connecting a typical lamp to a wall socket?* Presumably this will depend on the current through the wire—the bigger the current, the faster the electrons will move—and the current is the charge passing a point in the wire in a certain time:

$$I = \frac{Q}{t}.$$

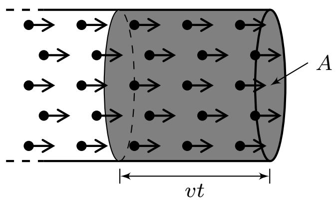


Figure 2: If the electrons are all moving with velocity $v$ along the wire, all of the ones in the shaded volume are going to emerge from the end in the next time $t$. Even the ones at the far left of this volume are going to make it to the end, as they will travel a distance $vt$ in this time. The volume that is shaded is therefore given by $V = Avt$, where $A$ is the cross sectional area of the wire.

In order to work out the charge $Q$, imagine counting the number $N$ of electrons that emerge from the end of the wire in a time $t$. The charge will then be $Q = Ne$, where $e$ is the charge on one electron ($-1.6 \times 10^{-19}$ C). Let's pretend that all of the electrons are moving along the wire with a speed $v$, meaning that the number $N$ which emerge from the end of the wire will be all of the electrons within a volume $V$ near the end of the wire. Considering figure 2, we can see that this volume will be $V = Avt$.

The current $I$ though the wire is thus

$$\begin{aligned} I &= \frac{Q}{t}, && \text{but since } Q = Ne, \\ &= \frac{Ne}{t}, && \text{but since } N = nV, \\ &= \frac{nVe}{t}, && \text{but since } V = Avt, \\ &= \frac{nAvte}{t}, && \text{and cancelling } t, \\ &= nAve, \end{aligned}$$

where $n$ is the number density of free electrons which we estimated earlier, $A$ is the cross sectional area of the wire, and $e$ is the charge on one electron ($-1.6 \times 10^{-19}$ C).

The velocity $v$ is known as the *drift velocity*, which is the velocity with which the electrons on average make progress along the wire when there is a current through it, for example when a battery has been connected across the ends of the wire.

The current through our typical lamp (which might have power 40 W) will be $I = P/V = 40\,\mathrm{W}/230\,\mathrm{V} = 0.2\,\mathrm{A}$. This gives the typical drift velocity of electrons along the wire (whose cross sectional area we can approximate with a square of side 1 mm using lumping again) as

$$\begin{aligned} v &= \frac{I}{nAe} \\ &\approx \frac{0.2\,\mathrm{A}}{10^{29}\,\mathrm{m}^{-3} \times 10^{-6}\,\mathrm{m}^2 \times 2 \times 10^{-19}\,\mathrm{C}} \\ &= 10^{-5}\,\mathrm{m\,s^{-1}}, \end{aligned}$$

which you might find surprisingly slow (even for a very large current—say 20 A—the drift velocity would only be of the order of $1\,\mathrm{mm\,s^{-1}}$).

The speed we should compare this with is the typical speed of a free electron in a metal,[2] which is around $1.6 \times 10^6\,\mathrm{m\,s^{-1}}$. The electrons behave a bit like particles in a gas, moving very rapidly indeed. All this whizzing about doesn't produce a flow of charge along a wire in a particular direction, though, because it is random and the electrons change direction when they hit the vibrating ions (which happens about every 450 Å at room temperature).

[2] This is the electron velocity at the Fermi surface [Source: Kittel, C., *Introduction to Solid State Physics* John Wiley and Sons, Inc. New York, 3rd ed., 1966].

*Estimate the time required for electrons to travel at this drift speed from the wall socket to the light bulb.* It would take over 100 000 hours to travel a metre along the wire at this speed (which may well be longer than the lifetime of the filament bulb!) Then how does the light bulb turn on right after you flip the switch? The free electrons start drifting through the wire within nanoseconds of closing the switch to turn on the light; this is because the 'electric field' that sets them drifting travels along the wire at almost the speed of light.