On capacitor charging

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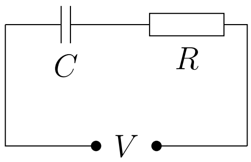
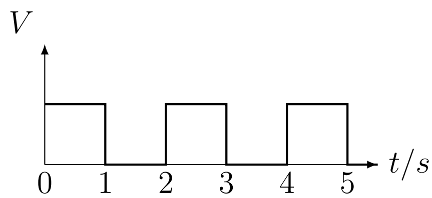
Warm-up problems

1. What is the formula for how the voltage across a capacitor which is being charged varies with time? Be very careful to show what each symbol means.
2. Find out what a *supercapacitor* is. What are its advantages over a battery in an application such as electric cars?

Regular problems

3. A capacitor of capacitance 200 µF is charged through a resistor of resistance 300 kΩ.
   (a) What is the time constant for this circuit?
   (b) How long will the capacitor take to charge to (i) 50%, (ii) 75%, (iii) 90% and (iv) 99.9% of a full charge?
4. (from Nelkon & Parker) The circuit below shows a capacitor C and a resistor R in series. The applied voltage varies with time as shown. The product CR is of the order 1 s. Sketch graphs showing the way the voltages across R and C vary with time. If the product CR were made considerably smaller than 1 s what would be the effect on the graphs?





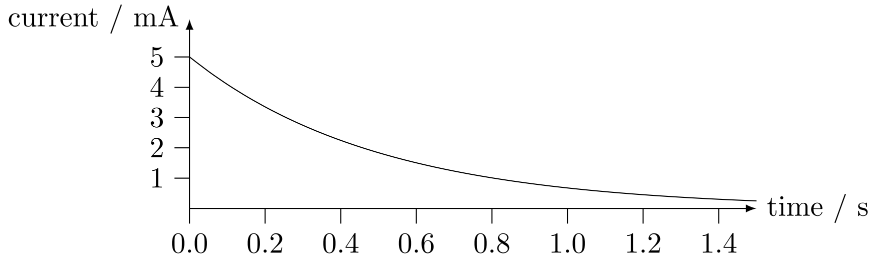
5. A 4.5 V battery pack is connected to a 220 µF capacitor through a resistance of 220 kΩ by a student wishing to investigate the charging of a capacitor.
   (a) What is the initial current?
   (b) What is the current after one minute?
   (c) What is the voltage after two minutes?
   (d) How long does it take for the voltage to get to (i) 3.0 V (ii) 4.0 V?
   (e) What is the current at each of these times?
   (f) How much energy is stored in the capacitor at each of these times?

6. Copy and complete the following table. If the graph is not going to be a straight line, write 'not straight' instead of the gradient (*Hint: you might like to try Isaac Physics A6*)

Equation	Plotted on y	Plotted on x	y-intercept	Gradient
$Q = Q_0(1 - e^{-t/RC})$	Q	t		
$V = V_0 e^{-t/RC}$	$\ln(V)$	t		
$Q = Q_0(1 - e^{-t/RC})$	Q	t		
$I = I_0 e^{-t/RC}$	I	$e^{t/RC}$		

7. Show that when a battery is used to charge a capacitor through a resistor, the heat dissipated in the resistor in the circuit is equal to the energy stored in the capacitor.

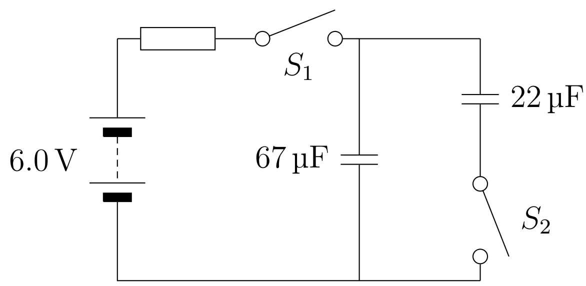
8. Examine the graph below, of the current through a capacitor.



   (a) Estimate the physical quantity corresponding to the area under the graph.
   (b) What was the initial current? Was the capacitor charging or discharging?
   (c) What is the time constant? If the resistor was 470 Ω, what was the capacitance?
   (d) What voltage is this capacitor charged up to?

Extension problems

9. (from CEA advanced extension award question) A feature of circuits containing charged capacitors is that there is often a spark at the contacts when a switch is opened or closed. In the circuit below the capacitors are initially uncharged and both switches are open.



Initially, switch S_1 is closed, charging the 67 µF capacitor from the battery. S_1 is then opened, leaving the 67 µF capacitor fully charged and the 22 µF capacitor still uncharged. When switch S_2 is closed, a spark occurs at the contacts of this switch. Estimate the energy dissipated in this spark. Is your value likely to be an over-estimate or an under-estimate? Give a reason.