Projectile Motion

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First, two problems!

Problem 1

A stone is dropped from rest from a 125 m high cliff. It accelerates under gravity with 10 m s^{-2}. How long does it take to hit the sea?

Problem 2

A railway truck is moving along a frictionless track at a constant speed of 20 m s^{-1}. How far does it travel in the time which you calculated in Problem 1?

Lesson Objectives

1. To understand the concept of projectiles
2. To be able to complete projectile calculations successfully

Specification Requirement

Independence of vertical and horizontal motion; problems will be soluble from first principles. The memorising of projectile equations is not required.

[AQA GCE AS and A Level Specification Physics A, 2009/10 onwards]

Another problem (to meet today's 2nd lesson objective!)

Projectile motion problem

A stone is thrown horizontally with velocity 20 m s^{-1} directly out to sea from the top of a cliff which is 125 m high. On leaving the shore, it undergoes projectile motion under the influence of gravity. Neglecting the effects of air resistance, how far out to sea will the stone create a splash, and how long will it take to do this?

[Take $g = 10$ m s^{-2}]

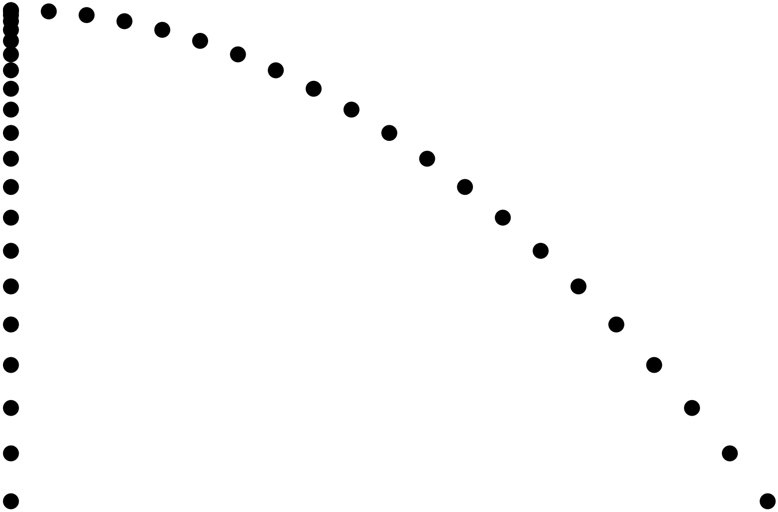
Starting points...

1. We need to consider vertical and horizontal components of motion. These are totally independent.
2. The acceleration involved is always g (downwards), which only affects the vertical component.
3. Any horizontal velocity is constant and unaffected by g.

A caution (health warning!)

TODAY THERE SHALL BE
NO AIR RESISTANCE

Separation of components



Vertical component (all vertical-only projectiles)

- Since we have a constant acceleration of g, we can use the SUVAT equations.
- If ‘upwards’ is taken as the increasing distance direction, the vertical velocity is given by

$$v = u - gt.$$

- We may write the displacement using another of the SUVAT forms:

$$y = ut - \frac{1}{2}gt^2.$$

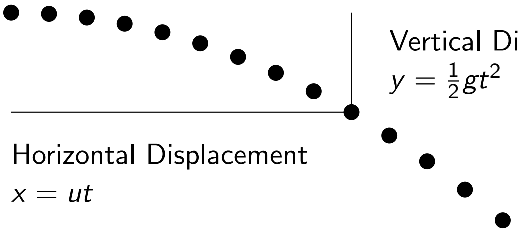
- In the example on the left, $u = 0$.

- The motion in the horizontal direction is simply linear motion at constant velocity.
- Since there is no component of g in the direction of the horizontal motion, there is no acceleration.
- By Newton's fist law, the body just carries on in this direction for ever (or until it hits something e.g. the ground!)
- The equations of motion are therefore fairly simple: the velocity is given by

  $$v = u,$$

  and the displacement is given by

  $$x = ut.$$



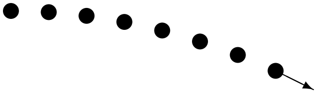
Object starts with perfectly horizontal motion at speed u.

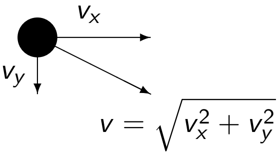
As it accelerates downwards, its path gets steeper and steeper.

The speed of projection, u, affects how far away the object lands. . .

. . . but not how long it stays in the air.

Finding the velocity





At any point, there are both horizontal and vertical components of the object's overall velocity:

$$v_x = u,$$

$$v_y = gt.$$

The combined speed is given by Pythagoras:

$$v^2 = v_x^2 + v_y^2$$
$$v = \sqrt{v_x^2 + v_y^2}.$$

An example

An object is projected at a horizontal speed of 15 m s^{-1} from the top of a 35 m tower. Calculate

1. how long it takes to reach the ground,
2. how far it travels horizontally,
3. its speed at impact.

Answers

1. 2.67 s
2. 40 m
3. 30.2 m s^{-1}