Newton's Laws 2

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First, an exam problem

Time allowed: 7 minutes
You will need your mechanics knowledge and to be able to apply Newton 1, 2, & 3.

Meanwhile, I shall collect your homework (**Newton's Laws 1**): please have it at the ready!
REMINDER: Office Hours are Tuesday (tonight) 3.45–5.0 p.m. in Room 20A (the science office).

Lesson Objectives

1. To quickly revise Newton's Laws of motion (done).
2. To understand terminal velocity using dimensional analysis.
3. To learn more about the acceleration due to gravity, g.

Textbook pp. 132–137

Acceleration due to gravity, g; detailed experimental methods of measuring g are not required.
Terminal speed.

A2 *terminal speed, Stokes' law for the viscous force on an oil droplet used to calculate the object radius*
$F = 6\pi\eta rv$

[AQA GCE AS and A Level Specification Physics A, 2009/10 onwards]

Fluid dynamics is a DRAG!

When an object moves in a fluid (liquid or gas), it experiences a resistance force called **drag**. This is the same force you feel when you stick your hand out of a car window into the breeze, or when you try to run in a swimming pool.

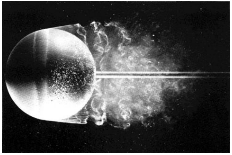


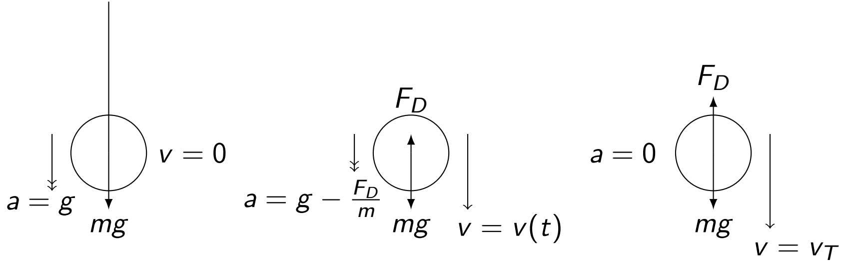
Image Credit: *An Album of Fluid Motion*, Van Dyke

In the last 2–3 lessons, we have for the most part ignored friction and air resistance. The reason is that these forces make life (i.e. the mathematics) much more difficult, and often we can get good enough results by neglecting them.

For example, when we considered an object which is dropped and falls under its own weight, we noted that its speed increases as it accelerates with g, giving it a speed at time t of $v = gt$.

Dropped stone with DRAG

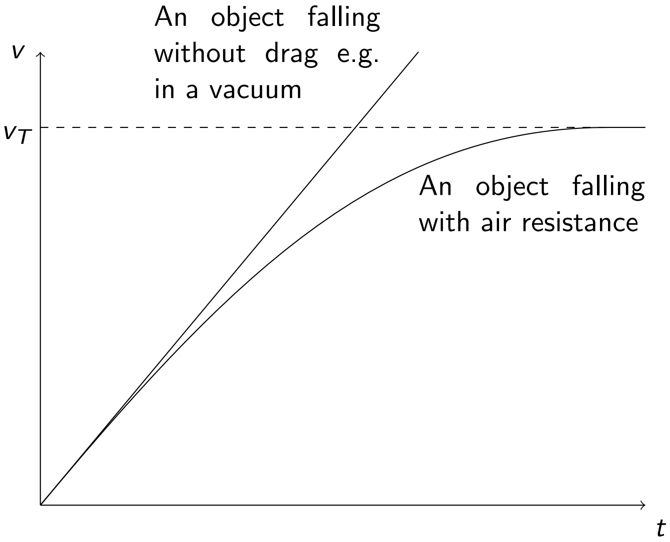
Without air resistance, it will continue to accelerate forever (or until it hits the ground!).
With air resistance, however, there will come a time when it will stop accelerating, and travel at a steady speed. This is because (Newton's first law) there is no net force acting on it—the air resistance is equal to its weight.
The steady speed which it reaches is known as its **terminal velocity**.



Start After short time Some time later

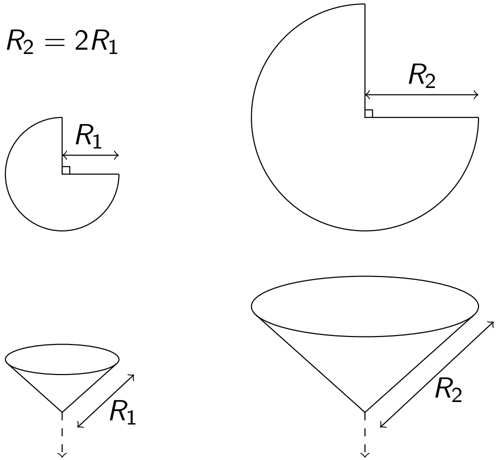
NB There is also a constant upward force of buoyancy, equal to the weight of the fluid displaced by the object, but I have chosen to ignore it here as it is

v–t graphs for motion with and without drag



Today's main problem

When you drop two paper cones of different size but the same shape, which one falls faster?



Have a guess!

We want to know the ratio $v_{\mathrm{big}}/v_{\mathrm{small}}$.
Your options are:

- $\frac{1}{4}$
- $\frac{1}{2}$
- 1
- 2
- 4

The exact approach is hopelessly difficult!

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}$$

$$\nabla \cdot \mathbf{v} = 0$$

Where

$\rho =$ air density

$p =$ pressure

$\mathbf{v} =$ velocity

$\nu =$ (kinematic) viscosity

$t =$ time

These are nasty, nonlinear, coupled, partial-differential equations. The top three equations are the Navier-Stokes equations of fluid dynamics, and the bottom equation is the continuity equation. In the solutions of the four equations is the answer to our question.

'Cheating': dimensional analysis

Variables that might determine the drag force F_D (NB The Units of a force are $[M][L][T]^{-2}$):

Var.	Units	Description
v	$[L][T]^{-1}$	velocity of falling object
ρ	$[M][L]^{-3}$	density of fluid (air)
A	$[L]^2$	area presented to fluid
ν	$[L]^2[T]^{-1}$	(kinematic) viscosity

Working it out

$$\underbrace{\text{drag force}}_{[\mathrm{M}][\mathrm{L}][\mathrm{T}]^{-2}} \sim \underbrace{\text{area}}_{[\mathrm{L}]^2} \times \underbrace{\text{density?speed?viscosity?}}_{[\mathrm{M}][\mathrm{L}]^{-1}[\mathrm{T}]^{-2}}$$

$$\frac{\text{kilogram} \times \text{meter}}{\text{second}^2} \qquad \text{meter}^2 \qquad \frac{\text{kilogram}}{\text{meter} \times \text{second}^2}$$

Working it out (2)

$$\underbrace{\text{drag force}}_{[M][L][T]^{-2}} \sim \underbrace{\text{area}}_{[L]^2} \times \underbrace{\text{density}}_{[M][L]^{-3}} \times \underbrace{\text{speed?viscosity?}}_{[L]^2[T]^{-2}}$$

$$\frac{\text{kilogram} \times \text{meter}}{\text{second}^2} \qquad \text{meter}^2 \qquad \frac{\text{kilogram}}{\text{meter}^3} \qquad \frac{\text{meter}^2}{\text{second}^2}$$

The answer...

$$F_D \sim \rho A v^2$$

$$\underbrace{\text{drag force}}_{[\mathrm{M}][\mathrm{L}][\mathrm{T}]^{-2}} \sim \underbrace{\text{area}}_{[\mathrm{L}]^2} \times \underbrace{\text{density}}_{[\mathrm{M}][\mathrm{L}]^{-3}} \times \underbrace{\text{speed}^2}_{[\mathrm{L}]^2[\mathrm{T}]^{-2}}$$

$$\frac{\text{kilogram} \times \text{meter}}{\text{second}^2} \qquad \text{meter}^2 \qquad \frac{\text{kilogram}}{\text{meter}^3} \qquad \frac{\text{meter}^2}{\text{second}^2}$$

Reality check!

$$F_D \sim \rho A v^2$$

Is this what we expect? Interestingly, since we found from our experiment that $F \propto A$ (there is no way that I know of to know this ahead of time), we were able to deduce that the drag force is *independent of the viscosity* for fast flows! This is fairly surprising I find...

We should (always) check our results for sanity by putting in some values. Reassuringly, our form for the drag force increases with velocity, and is zero where $v = 0$. We also expect the drag force to increase with the density of the fluid (experience should tell you that it's easier to run through air than water!)

Follow-up question 1: to test **your** understanding...

In the first experiment, we raced two cones, one of which was twice as large, giving a factor of four in the area and mass. Now we race one small cone against four (stacked) small cones. What will be the terminal velocity ratio v_4/v_1?

The options are the same as before:

- $\frac{1}{4}$
- $\frac{1}{2}$
- 1
- 2
- 4

Follow-up question 2: to test **your** understanding...

Our experiment was for 'fast' flows[1], but if we were to repeat it for 'slow' flows[2] (we probably don't have time in the lesson to do this), we should find that

$$F_D \sim R,$$

where R is the size of the object (with dimensions of [L]).

Challenge

Can you work out the form of the drag force F_D for viscous flows?

[1] Fast compared to what? Well, strictly I mean flows in the turbulent limit, or at high Reynolds number ($Re \approx 10^4$). Most fluid flow–large raindrops falling in air, ships traveling in water, cyclists racing in air–is turbulent.

[2] These are flows in the viscous limit, or at low Reynolds numbers e.g. a marble falling down a tube of glycerine at $Re \approx 0.1$.

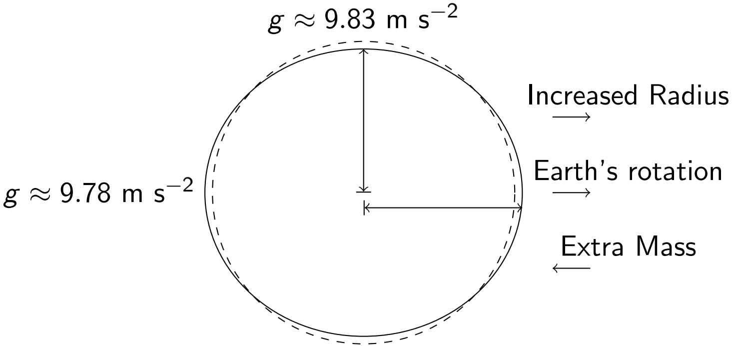
Our familiar friend... g

Using Newton's law of gravitation $F = \frac{Gm_1 m_2}{r^2}$ and his second law of motion $F = mg$, we can work out the gravitational acceleration at the surface of the Earth:

$$g = \frac{GM_E}{R_E^2}.$$

Is this a constant?

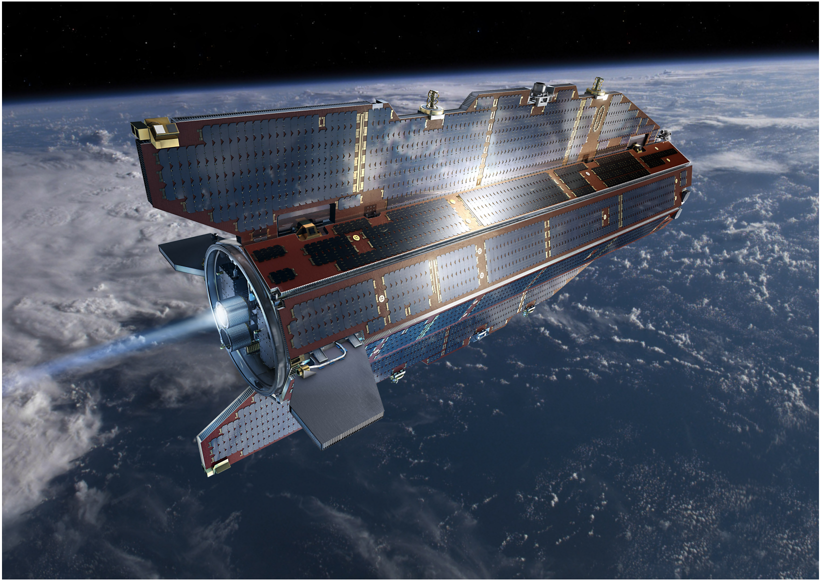
Variations in g



Measuring g: use a gravimeter!

- Uses free fall to measure g
- Vacuum chamber to eliminate drag
- Speed of retroreflector measured using laser
- Transparent disc dropped below reflector

ESA GOCE satellite



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31 March 2011 Last updated at 16:27
Gravity satellite yields 'Potato Earth' view
By Jonathan Amos
Science correspondent, BBC News, Munich
It looks like a giant potato in space.
And yet, the information in this model is the sharpest view we have of how gravity varies across the Earth.
The globe has been released by the team working on Europe's Goce satellite.
It is a highly exaggerated rendering, but it neatly illustrates how the tug we feel from the mass of rock under our feet is not the same in every location.
Gravity is strongest in yellow areas; it is weakest in blue ones.
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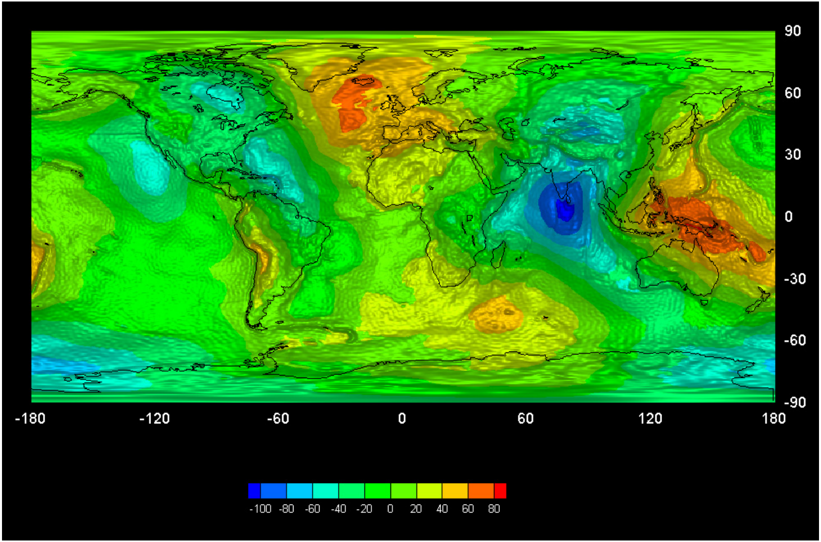

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