On moments

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Warm-up problems

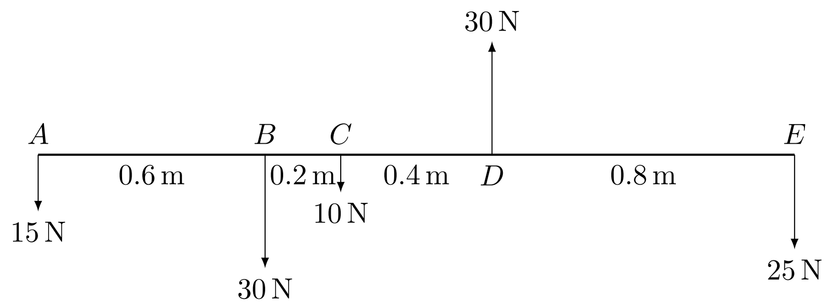
1. What is a ‘moment’ in physics, and how might you work one out? Draw a diagram to illustrate your answer.
   A moment is the turning effect of a force about some point, given by the product of the force and the perpendicular distance of the line of action of the force from the point.

2. What does *equilibrium* mean, and what are the conditions for an object to be in equilibrium?
   If an object is in equilibrium, its motion is not changing. This means that its centre of mass is not accelerating (though it may be moving at a constant speed in a straight line) and it is not changing its speed of rotation. This means that the sum of all the forces on it must be zero (Newton’s first law), and also the sum of the clockwise and anticlockwise moments must be zero.

3. What is the *centre of mass* of an object, and how does the hanging method for finding it used in class work?
   An object’s centre of mass is the point where all of its mass can be considered to be concentrated at a point when we are modelling the object as a point particle. It is a bit like an average position for where the mass of the object is located. The hanging method works because, when the object hangs in equilibrium so it is still, the centre of mass must not exert a turning moment on the pivot point. This means it hangs so the centre of mass is directly below the pivot, establishing a line on which the centre of mass must be. Finding the crossing position of two or more such lines gives the centre of mass.

Regular problems

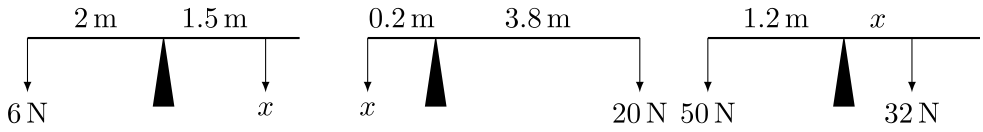
4. In the following diagram, the beam has negligible mass (so the weight force on it can be ignored).



   (a) Find the total moment of all the forces about the points A, B, C, D and E.
   A ($\circlearrowright$): $[15\,\text{N} \times 0\,\text{m}] + 30\,\text{N} \times 0.6\,\text{m} + 10\,\text{N} \times 0.8\,\text{m} - 30\,\text{N} \times 1.2\,\text{m} + 25\,\text{N} \times 2.0\,\text{m} = 40\,\text{N}\,\text{m} \circlearrowright$
   B ($\circlearrowright$): $-15\,\text{N} \times 0.6\,\text{m} + [30\,\text{N} \times 0\,\text{m}] + 10\,\text{N} \times 0.2\,\text{m} - 30\,\text{N} \times 0.6\,\text{m} + 25\,\text{N} \times 1.4\,\text{m} = 10\,\text{N}\,\text{m} \circlearrowright$
   C ($\circlearrowright$): $-15\,\text{N} \times 0.8\,\text{m} - 30\,\text{N} \times 0.2\,\text{m} + [10\,\text{N} \times 0\,\text{m}] - 30\,\text{N} \times 0.4\,\text{m} + 25\,\text{N} \times 1.2\,\text{m} = 0\,\text{N}\,\text{m}$
   D ($\circlearrowright$): $-15\,\text{N} \times 1.2\,\text{m} - 30\,\text{N} \times 0.6\,\text{m} - 10\,\text{N} \times 0.4\,\text{m} + [30\,\text{N} \times 0\,\text{m}] + 25\,\text{N} \times 0.8\,\text{m} = -20\,\text{N}\,\text{m} \circlearrowright$
   E ($\circlearrowright$): $-15\,\text{N} \times 2.0\,\text{m} - 30\,\text{N} \times 1.4\,\text{m} - 10\,\text{N} \times 1.2\,\text{m} + 30\,\text{N} \times 0.8\,\text{m} + [25\,\text{N} \times 0\,\text{m}] = -60\,\text{N}\,\text{m} \circlearrowright$

(b) At which of these points could you place a pivot to balance the beam and what would the force on the pivot be?
The only point where you could place a pivot with the clockwise and anticlockwise moments being equal (so it will be balanced in equilibrium) is at C. If a pivot is placed there, it would have a force down on it, and it would need to provide an upward force on the beam (otherwise the beam would fall). This force would balance (and therefore be equal to) the overall downward force of $15\,\mathrm{N} + 30\,\mathrm{N} + 10\,\mathrm{N} - 30\,\mathrm{N} + 25\,\mathrm{N} = 50\,\mathrm{N}$.

item In all the situations below, the beams (again of negligible mass) are in equilibrium. Find x in each case and give its unit.



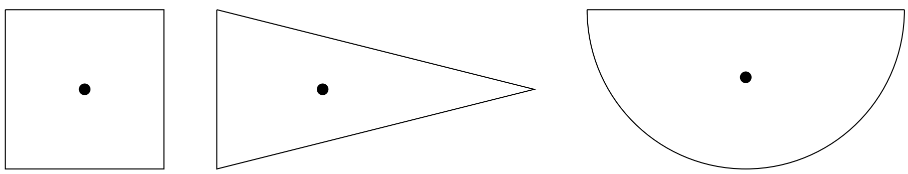
$$\circlearrowleft = \circlearrowright$$
$$6\,\mathrm{N} \times 2\,\mathrm{m} = x \times 1.5\,\mathrm{m}$$
$$\frac{6\,\mathrm{N} \times 2\,\mathrm{m}}{1.5\,\mathrm{m}} = x$$
$$x = 8\,\mathrm{N}.$$

$$\circlearrowleft = \circlearrowright$$
$$x \times 0.2\,\mathrm{m} = 20\,\mathrm{N} \times 3.8\,\mathrm{m}$$
$$x = \frac{20\,\mathrm{N} \times 3.8\,\mathrm{m}}{0.2\,\mathrm{m}}$$
$$x = 380\,\mathrm{N}.$$

$$\circlearrowleft = \circlearrowright$$
$$50\,\mathrm{N} \times 1.2\,\mathrm{m} = 32\,\mathrm{N} \times x$$
$$\frac{50\,\mathrm{N} \times 1.2\,\mathrm{m}}{32\,\mathrm{N}} = x$$
$$x = 1.9\,\mathrm{m}.$$

5. In the last question, why do you take moments about that particular point (*Hint: think about the vertical equilibrium of forces.*)
The reason it is easiest to take moments about the pivot each time is that this means we don't need to worry about the upward force from the pivot (which we may or may not be able to work out, depending on whether we are given the forces).

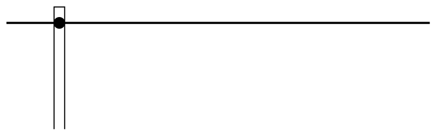
6. Draw the shapes below and show where you think the centre of mass is, with a short comment why you think this.



7. Explain why it is difficult to balance a metre ruler so that it is vertical on top of your finger, and how a traffic cone's design makes it difficult to knock over.
For an object not to be balanced, its centre of mass must provide a moment which topples it, i.e. the centre of mass is not above its base.
A metre ruler is very hard to balance vertically on your finger because its centre of mass is about 50 cm above the finger, and the base is very small. A very small angle from the vertical will make the ruler start to fall over as the centre of mass will have a moment about the base.
A traffic cone, on the other hand, has a wide base and a low centre of mass. It would have to be tipped to a very high angle for the moment to not be above the base. In fact, it would probably slide first unless it was on a surface with very high friction.

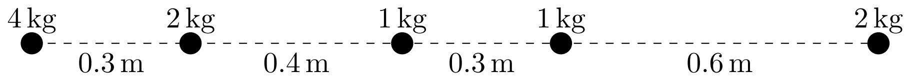
Extension problems

9. Suppose a long uniform metal pole is pivoted across a gateway a short way from its end. Can you design a method to find the mass of the pole, even if it can't be removed from the pivot to weigh it?



We could use a known masses to provide a known weight force on the end of the pole nearest the pivot so that it is in equilibrium. We could then measure the distance from the known mass to the pivot and the length of the pole. Knowing that the centre of mass of the pole is halfway along the beam, we can use the fact that the clockwise moment equals the anticlockwise moment when the pole is balanced, we could then work out the unknown mass of the pole without removing it from the pivot.

10. How is the center of mass like an average (mean)? Can you use this to find the centre of mass of the following masses?



The centre of mass of an object is very like a mean position of the mass in the object. If there is more mass at a position, it counts for more in the mean weighting. To work out the centre of mass of these objects, we can work out their mean position, weighted by their masses:

$$\frac{[0\,\mathrm{m} + 0\,\mathrm{m} + 0\,\mathrm{m} + 0\,\mathrm{m}] + (0.3\,\mathrm{m} + 0.3\,\mathrm{m}) + 0.7\,\mathrm{m} + 1.0\,\mathrm{m} + (1.6\,\mathrm{m} + 1.6\,\mathrm{m})}{10} = 0.55\,\mathrm{m}$$