10A2: Parallax

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1 What is parallax?

When we look out of a car window, trees, chimneys, towers and other objects appear to be moving relatively to each other. A tree may appear to be to the right of a church spire and a few seconds later to the left of it. Their actual positions in space have, of course, remained fixed. Their apparent relative movement of objects when you observe them from different locations is known as *parallax* (the word comes from the ancient Greek $\pi\alpha\rho\alpha\lambda\lambda\alpha\xi\iota\varsigma$—parallaxis—meaning 'alteration').

Nearby objects have a larger parallax than more distant objects when observed from different positions, so parallax can be used to determine distances.

2 Observing parallax

A simple way to observe parallax is to close one eye and point at a distant object. If you then swap eyes, your finger will appear to move, and will no longer point at the object. This happens because our eyes are separated by about 6 cm, and so they each get a slightly different view of the world from their different positions. This is important, since it enables us to judge distance—this is called depth perception—and in fact 3D cinema uses various techniques to show each of our eyes slightly different images, with a parallax between them, enabling things to be viewed in real space rather than the usual flattened picture.

Another simple everyday example of parallax in a car can be seen in cars which use an older-style 'needle' speedometer gauge. When viewed from directly in front, the speed may show exactly 60 mph; but when viewed from the passenger seat the needle may appear to show a slightly different (higher!) speed, due to the angle of viewing. In physics, this can occur with all sorts of measuring instruments, leading to errors in the readings known as *parallax errors*. Always read from directly in front of the needle!

3 Measuring distance

As we have already noted, the parallax information that you get from looking at an object from two different positions should be enough to work out how far away the object is, **without** actually having to go there.

3.1 The thumb method

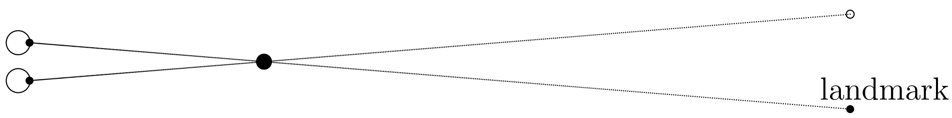
Here is a useful method for getting a rough idea of how far away something is. Suppose you want to estimate the distance to some distant landmark, e.g. a water-tower, tree or building.

1. Stretch your arm forward and, with one eye closed, move your thumb (making a 'thumbs-up') so that you see your thumb covering the landmark.

2. Now open the eye you had closed, and close your other eye, without moving your thumb. It will now appear that your thumb has moved: it will no longer be at the landmark, but will be in front of some other point at the same distance.

3. Guess the distance from this point to your landmark, by using the estimated heights of trees, widths of buildings, distances between power line poles, lengths of cars &c.

4. The distance to the landmark is **ten times** the distance you have guessed.



Why does this work? Although people vary in size, the proportions of the average human body are fairly constant, and for most people, the angle in the triangle made between the eyes and the outstretched thumb is about $6°$ – this is the angle of parallax of your thumb, viewed from your eyes. This means that the height of this (isosceles) triangle—the length of your arm—is 10 times its base—the distance between your eyes—and this triangle has the same proportions (is mathematically similar) to the much larger triangle formed by your thumb, the landmark, and the point your thumb seemed to move to. This means the distance to the landmark is also 10 times the distance you guessed in 3.

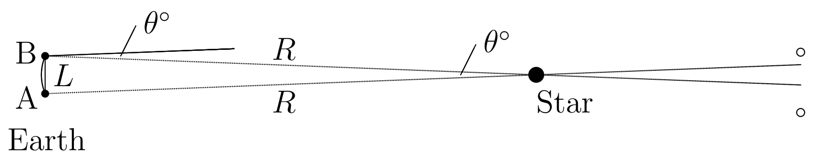
3.2 Distances to stars

Astronomers use parallax to measure the distances to stars. As we know, near objects have a larger parallax angle than faraway ones, and for objects beyond a certain distance, the angles involved are simply too small to measure, meaning that this method is only good for nearby stars.[1] To measure the distances to further away stars, methods based on star brightness or redshift are used.[2]

Of course, stars don't appear to shift when we view them from different eyes! Astronomers need to use two points which are as far apart as possible, to observe a shift at all. The largest baseline available for astronomers is the diameter of the Earth's orbit, some 300 000 000 km. Astronomers use telescopes to look at the star twice half a year apart, to see whether it has shifted its position in the sky against the much more distant background stars.

Then the distance to the star can be worked out. The method we use was already known to the ancient Greeks more than 2000 years ago. They knew the circumference of a circle of radius r is $2\pi r$, where $\pi = 3.14159\ldots$

Imagine drawing a circle which has the star at its centre, which passes through the Earth's summer and winter positions (A and B on the diagram). Since the angle θ of this sector of the whole circle is so small, the length of the straight-line baseline L is not much different from the length of the arc of the circle AB. In this method, we make the approximation that these two lengths are the same.



The length of a the circle's arc is a fraction of the entire circumference, just as the angle $\theta°$ is a fraction of the whole circle's 360°:

$$L \text{ covers an angle } \theta°$$

$$2\pi R \text{ covers an angle } 360°.$$

From this, we can work out R, the distance from the Earth to the star. Since

$$\frac{L}{2\pi R} = \frac{\theta}{360°},$$

$$R = \frac{360°}{\theta°} \times \frac{L}{2\pi}.$$

How much do the stars shift when viewed from two points 300 000 000 km apart? Actually, very, very little. Even though astronomers knew that they ought to be able to observe the parallax shifts, none were observed for many years, and it was only in 1838 that definite parallaxes were measured for

[1] Up to about 1,600 light-years away at the moment, which is only a little more than one percent of the diameter of our Milky Way galaxy.

[2] See `http://en.wikipedia.org/wiki/Cosmic_distance_ladder` for further information.

some of the nearest stars. The reason for this is that the parallax angles are *tiny*.

Such measurements demand enormous precision. Where a circle is divided into 360°, each degree is further divided into 60 minutes (60′)—also called 'minutes of arc', to distinguish them from 'minutes of time'—and each minute contains 60 seconds of arc (60″). All observed parallaxes are less than 1″, which is at the limits of the resolving power of telescopes.

In measuring star distances, astronomers frequently use the **parsec**, the distance to a star whose annual parallax is 1″ – one second of arc. One parsec equals 3.26 light years, but no star is this close to us: the nearest is Proxima Centauri, with a distance of 4.24 light years[3] and a parallax of 0.75″ (this is the same as the angle between the edges of a 2p coin which is 5.3 km away!)

4 Problems

1. The baseline used in measuring parallaxes of nearby stars is the diameter of the earth's orbit (2 AU = 300 000 000 km).
   If the parallax angle measured is 0.00015°, how far away is the star?

2. Proxima Centauri is the nearest star to us (after the Sun). It is 4.24 light years away.
   What would we measure its parallax angle to be?

References

http://en.wikipedia.org/wiki/Parallax
ABBOTT, A.F. *Ordinary Level Physics*, 4th Edtn., 1984, pp. 228
http://www-spof.gsfc.nasa.gov/stargaze/Sparalax.htm

[3]A light year is the distance which light, moving at 300 000 km/s, covers in a year. The sun is eight light minutes away, Pluto about 5 light hours.