BISHOP HEBER
HIGH SCHOOL

Nuclear decay

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Exam question

Lesson Objectives

1. To appreciate the inherent probabilistic (quantum) nature of nuclear decay.
2. To know what half life is and be able to determine it from graphical data.
3. To know and use the decay equations on the specification.

Textbook pp. 162–167

REMINDER: Office hours are week 1 Tuesdays 3.45–5.0 p.m. in Rm. 19.
Next office hours: Tuesday 12 March 2013

Specification Requirement

Radioactive decay

Random nature of radioactive decay; constant decay probability of a given nucleus;

$\frac{\Delta N}{\Delta t} = -\lambda N$, $N = N_0 e^{-\lambda t}$

Use of activity $A = \lambda N$

Half life, $T_{1/2} = \frac{\ln 2}{\lambda}$*; determination from graphical decay data including decay curves and log graphs; applications e.g. relevance to storage of radioactive waste, radioactive dating.*

[AQA GCE AS and A Level Specification Physics A, 2009/10 onwards]

Spontaneous, random decay

- Radioactive nuclei decay spontaneously, the process cannot be speeded up or slowed down. In particular it is not affected by:
  1. chemical combination
  2. changes in physical environment
- There is no way of predicting when a particular nucleus will decay, or which of a collection of nuclei will decay next. It is genuinely random, because we cannot know ahead of time what will happen.
- The probability of a particular nucleus of an isotope decaying in a certain time is constant for that isotope.

The end of determinism

Before 1900, physicists thought that, from knowing the initial conditions of a situation (and the laws of physics!) you could work out everything that would subsequently happen. This was called *determinism*.

- In 1900, Lord Kelvin famously said "there is nothing new to be discovered in physics. All that remains is more and more precise measurement."
- On 14 December that same year, Max Planck published a paper introducing *quanta*, and started the revolutionary new theory of quantum mechanics.

Radioactivity turned out to be the first truly random, *probabilistic* process.

The end of determinism

There are many more.

- e.g. We can't predict when an excited atom will return to its ground state and emit a photon.
- e.g. Maybe a piece of glass reflects 94% of light. So 94 photons out of every 100 are reflected and 4 are transmitted. We can't tell which 4 ahead of time!

Quantum theory tells us that the most fundamental events are random: we can only ever know the probabilities for various outcomes!

The rise of ‘probabilism’

- The quantum world is inherently probabilistic.
- Einstein hated this: “God does not play dice!”
- It turns out that, even though physics has ‘retreated’ from attempting to predict everything, knowing the probabilities it still enormously useful.
- So long as you don’t ask questions like ‘How does the atom know to decay *then*’, or ‘How does *that* photon know to reflect’, the outcomes of experiments can be predicted very accurately indeed (*positivism*).
- Quantum theory is (probably) the most successful and accurate physical theory ever.

Making predictions from probabilities

As we've seen, all we know is that a given nucleus will have a constant decay probability. How can we use this to make useful predictions?

- Since atoms are very small, normally we deal with (very) large numbers of nuclei!
- So statistics works, and will help us to work out what will happen.
- For large numbers of nuclei, the proportion of nuclei that decay in a certain time will be constant, e.g. if 80% decay in 30 s (leaving 20% undecayed), then in the next 30 s a further 80% will decay (leaving only 4% undecayed).

Define 'half-life'.

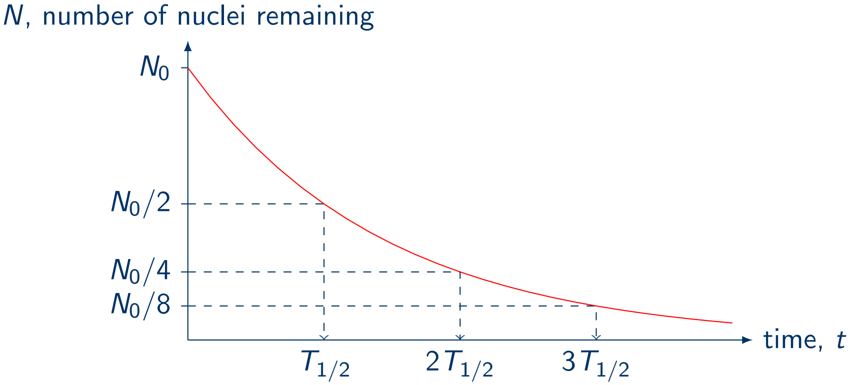
How does number of nuclei remaining depend on time?

Half-life

The half-life $T_{1/2}$ of a radioactive nuclide is the time taken for half of the nuclei of that nuclide to decay.

- It isn't useful to think of a 'life'—the time that elapses before all the nuclei have decayed—as this may be unpredicatably short/long!
- $T_{1/2}$ is a constant for the nuclide. It doesn't depend on the number of nuclei present.
- Half-lives have a very wide range of values: e.g. $^{99}\mathrm{Kr}$, 13 ms; $^{238}_{92}\mathrm{U}$, 4.51×10^9 year.
- This gives rise to a characteristic decay curve

Exponential decay



Where have you seen this before?

Exponential decay

As well as in radioactive decay of a nuclide, exponential decay is found in:

- capacitor discharge
- slow heating/cooling (Newton's law of cooling)
- rolling many dice with removal each time (the radioactive analogue experiment we did)
- chemical reactions (first-order, rate depends on concentration of one reactant only)
- water emptying from a tube: height falls exponentially as rate of flow depends on height remaining
- absorption of light by a substance (Lambert-Beer law)
- overdamping of an oscillation

. . . and many more!

How can we measure the activity of a sample?

Activity

The activity A of a sample is the number of nuclei which decay per second. This was given a special SI unit (rather that s^{-1} or Hz) for safety reasons: the becquerel (Bq) is only used for radioactivity.

$$1\,\mathrm{Bq} = 1\,\mathrm{s}^{-1}$$

- the activity depends on the mass of a sample
- the activity decreases with time as the sample decays
- the Bq is a small unit, so in industry/physics the Curie Ci is used instead ($1\,\mathrm{Ci} = 3.7 \times 10^{10}\,\mathrm{Bq}$).

Discovering the curve's equation

If N is the number of nuclei in a sample of a particular nuclide at any given time, then the rate of decay (change in N over time taken for the change: this is the activity A of the particular nuclide) is proportional to the number of nuclei present,

$$\frac{\Delta N}{\Delta t} \propto N, \text{ or}$$

$$\frac{\Delta N}{\Delta t} = -\lambda N,$$

where λ is a constant of proportionality called the *decay constant*. Since N decreases as t increases, a minus sign is included in the equation so that λ is a positive constant. What is its unit?

Discovering the curve's equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N$$

This already tells us some things about the curve, e.g. as N halves, the gradient halves (note I've put ds instead of nasty Δs!)

Can you get the curve's equation (integrate!) in the form $N = f(t)$?

Discovering the curve's equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N$$

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$$\frac{1}{N}\mathrm{d}N = -\lambda \mathrm{d}t$$

$$\int \frac{1}{N}\mathrm{d}N = -\lambda \int \mathrm{d}t$$

$$\ln N = -\lambda t + \text{const.}$$

$$N = e^{\text{const.}} e^{-\lambda t}$$

When $t = 0$, the number of nuclei is the initial number N_0, i.e.

$$N = N_0 e^{-\lambda t}$$

Relating back to half-life

$$N = N_0 e^{-\lambda t}$$

If the half-life is represented by $T_{1/2}$, this is when N has fallen to $N_0/2$, so

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}},$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}},$$

$$2 = e^{\lambda T_{1/2}},$$

$$\ln 2 = \lambda T_{1/2},$$

$$T_{1/2} = \frac{\ln 2}{\lambda}.$$