Density

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'What weighs more: a pound (lb) of feathers or a pound of lead?' Of course, we all know that they both weigh the same (because they have the same mass), but we often hear the expressions 'as light as a feather' or 'as heavy as lead', and it is true that equal volumes of different substances vary considerably in mass. In physics, we use the property of *density* to compare the 'lightness' or 'heaviness' of different materials.

Definition

The density of a material is its mass per unit volume.

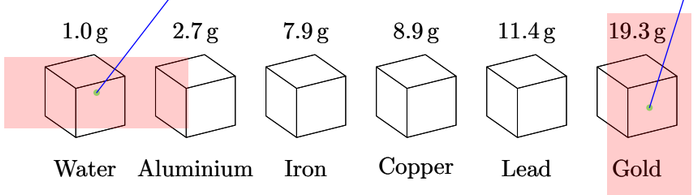
$$\text{density} = \frac{\text{mass}}{\text{volume}}$$
$$\rho = \frac{m}{V}$$

The SI unit of density is the kilogram per cubic metre, $\mathrm{kg\,m^{-3}}$, for which we must use kilograms to measure the mass and cubic metres to measure the volume. Sometimes it is more convenient to measure the density in different units, most commonly g for mass and cm^3 for volume. It is very easy to convert between the two, and it is worth remembering that water has a density of $1\,\mathrm{g\,cm^{-3}}$:

$$\text{density of water} = 1\frac{\mathrm{g}}{\mathrm{cm}^3} = \frac{1/1000\,\mathrm{kg}}{(1/100\,\mathrm{m})^3} = 1000\,\mathrm{kg\,m^{-3}}.$$

Typical densities

We have already seen that a cubic centimeter of water will have a mass of 1 g. Let us see what mass a cm^3 of various metals will have:



What factors affect density? (e.g. temperature)

How is 1g/cm3 equal to 1000kg/m3? (Jamie Ganley)

Are all these cubes of materials at the same temperature? (Jamie Ganley)

Why does gold have a higher density than lead when lead has more nucleons? (Ciaran O'Shea)

The density tells us how well the matter is packed together in a material. It depends on the mass of the particles that make up the material, and how efficiently they are packed together. A perfect vacuum has a density of zero[1], gases have low densities since their particles are far apart, and solids have much higher densities. The densities of most solids lie in the range 500 to 1000 $\mathrm{kg\,m^{-3}}$, and gases in general are about 10^3 times less dense than this.

substance	density / $\mathrm{kg\,m^{-3}}$
lead	11 300
aluminium	2700
water	1000
air	1.2
hydrogen	0.08

Even in a solid, though, the atoms are almost entirely empty space, with almost all of the mass contained in the protons and neutrons at the centre of the atom (the electrons have a vanishingly small mass in comparison). Nuclear material therefore has incredibly high density. If we could somehow collapse everyday objects by removing the empty space in their atoms, they would still have the same mass, but would occupy a tiny fraction of their former volumes. Careful observations of certain white dwarf stars (such as Sirius B) have led astronomers to calculate that such a 'superdense' state[2] can exist; a teaspoon of Sirius B would weigh not grams but tonnes!

Measuring density

The densities of many substances have been carefully measured, and compiled in tables of physical constants, and an unknown substance can often be identified by measuring its density and comparing it with known substances. In order to determine the density, usually the mass and the volume must be measured first. The mass can be very accurately measured in the laboratory using a balance.

The volume can be more of a challenge. If the material is a solid in a regular shape, these can be accurately measured, e.g. with a vernier caliper, and the volume worked out. Otherwise the solid could be put into a measuring cylinder partly filled

[1] At least classically: quantum theory tells us that there is some 'vaccum energy density' due to *ground state fluctuations*, as shown by the Casimir effect.

[2] The protons and neutrons of many atoms are packed close together, surrounded by an electron gas. This is known as *electron-degenerate matter*. More massive stars can enter a denser state still, where the protons become bonded to the electrons, forming a *neutron star* (observationally confirmed as pulsars).

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Would temperature of a substance affect its density? (Ciaran O'Shea)

Didn't know a vacuum has density of zero... (David Marlow)

Didn't know the density of solids is in the range 500-1000 kg/m3 (David Marlow)

Why does nuclear material have high density? (Jacky Chung)

How do we know the density of Sirius B so accurately? (Ciaran O'Shea)

Where/what is Siruis B?

What causes the superdensity on stars? (Jamie Ganley)

Does density ever fluctuate randomly? (Jamie Ganley)

Why are there ground state fluctuations?

What's the Casimir effect? (Jacky Chung)

Is there more than one electron gas that can surround the nucleons?

Do we need to know about ground state fluctuations / electron degenerate matter?

with water and the rise in water could be noted. For a liquid, a burette of known volume could be filled with the liquid, and then that volume could be weighed.

History

The most famous and oldest story of identification by density is that of Archimedes and the King of Sicily's crown. Archimedes was a great scientist of ancient times, who lived in Syracuse in about 300 B.C. The King of Sicily commissioned his royal goldsmith to make him a gold crown from a lump of gold. When the crown had been made, though its mass was correct, the King suspected the goldsmith of cheating him by including some silver, a much cheaper metal, in the crown and stealing some of the gold. Archimedes had to find out whether this had indeed happened, without damaging the crown.

He was unable to solve this problem until one day, on getting into his bath, he noticed that the water rose. Whereupon, he realized how to solve the problem—presumably by obtaining lumps of gold and silver of the same mass as the crown, measuring their volumes by seeing how much the water rose in a container, and comparing these with the volume of the crown, thus comparing the density of the three objects, allowing him to calculate the crown's composition and tell the King whether he had been robbed—and he celebrated by rushing naked through the streets of Syracuse shouting '$\epsilon\upsilon\rho\eta\kappa\alpha$ / eureka!' (which means 'I have found it!')

Some uses of density

An engineer can work out the weight of a bridge from its dimensions, using the density of the materials used in its construction. Aircraft are made from aluminium alloys, which are as strong as steel, but which, volume for volume, weight less than half as much.

A *hydrometer* can be used to accurately measure the density of a liquid within a narrow range, and this can be used to determine its alcohol content, e.g. in beer and wine making. This method is also used by revenue officers to work out how much tax must be paid on spirits.

A ship often has a scale near its waterline which works on the same principle, allowing it to be loaded in freshwater ($\rho = 1000\,\mathrm{kg\,m^{-3}}$, and then float at the correct draft to maintain a safe bouyancy in sea water ($\rho = 1025\,\mathrm{kg\,m^{-3}}$).

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Fun fact for you, sir: Eureka is pronounced "evrika" in Greek (Ciaran O'Shea)

How accurate can the bridge weight be? (David Marlow)

What is a hydrometer?

What is a hydrometer? (Jamie Ganley)

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